Advanced Macroeconomics

The Solow Model

Stylised Facts about Growth

- Growth rates of per capita GDP are not constant over time. It is only in the past 200 years that the world economy has shown sustained rates of growth
- Growth rates of per capita GDP vary a lot across countries, with no particular correlation between growth and GDP per capita
- This correlation does hold reasonably well, however, for OECD nations
- For the US, the real return to capital, the capital to output ratio, the growth rate of GDP per worker, and the shares of income paid to labor and capital are all roughly constant over long periods of time
- There is a strong positive correlation between growth and international trade

Solow Model Dynamics

- The equilibrium steady-state outcome is determined by the intersection of the savings curve $sf(k_t)$ and the break-even investment line $(n+g+\delta)k$
- The Solow-Swan economy is globally stable
- Any change in α will only affect the savings curve. To determine how, differentiate $sf(k_t)$ with respect to α and observe how change depends upon the level of capital
- The golden rule level of k maximizes the steady state consumption per unit of effective labour, relative to the savings rate

Solow Model Implications

- Output per worker in major industrialized countries today is 10 times larger than it was 100 years ago, and 10 times larger than it is in poor countries. To account for this on the basis of differences in capital, capital per worker must differ by a factor of 1000. However, there is no evidence of such differences in capital stocks.
- Prediction of convergence is not borne out by the data
- Limited use as most of the interesting aspects, such as savings and technology, are exogenous
- Some testable comparative static properties are broadly consistent with the data: Output per worker is positively related to the saving rate s and negatively related to the rate of population growth n

Growth Accounting

$$\begin{aligned}
\frac{dY}{dt} &= \frac{dF(K_t, E_t L_t)}{dt} \\
&= \frac{\partial Y}{\partial K}\left(\frac{dK}{dt}\right) + \frac{\partial Y}{\partial L}\left(\frac{dL}{dt}\right) + \frac{\partial Y}{\partial E}\left(\frac{dE}{dt}\right) \\
\frac{dY/dt}{Y} &= \frac{1}{Y}\frac{\partial Y}{\partial K}\left(\frac{dK}{dt}\right)\left(\frac{K}{K}\right) + \frac{1}{Y}\frac{\partial Y}{\partial L}\left(\frac{dL}{dt}\right)\left(\frac{L}{L}\right) + \frac{1}{Y}\frac{\partial Y}{\partial E}\left(\frac{dE}{dt}\right)\left(\frac{E}{E}\right) \\
&= \frac{K}{Y}\frac{\partial Y}{\partial K}\frac{(dK/dt)}{K} + \frac{L}{Y}\frac{\partial Y}{\partial L}\frac{(dL/dt)}{L} + \frac{E}{Y}\frac{\partial Y}{\partial E}\frac{(dE/dt)}{E}
\end{aligned}$$

$$gr(Y) = \frac{K}{Y}\frac{\partial Y}{\partial K} gr(K) \ + \frac{L}{Y}\frac{\partial Y}{\partial L} gr(L) + \frac{E}{Y}\frac{\partial Y}{\partial E} gr(E)$$

In competitive market $\frac{\partial Y}{\partial K} = r$ and $\frac{\partial Y}{\partial L} = w$. K, Y, L and their growth rates can be measured directly.

$$gr(Y) = \frac{K}{Y} r gr(K) + \frac{L}{Y} w gr(L) + \frac{E}{Y}\frac{\partial Y}{\partial E} gr(E)$$
$$gr(Y) - \frac{K}{Y} r gr(K) - \frac{L}{Y} w gr(L) = \frac{E}{Y}\frac{\partial Y}{\partial E} gr(E) = Solow\ Residual$$

The Diamond Model

Setup

- Economy populated by 2-period-lived individuals
- Population growth rate of n
- Each young individual supplies a unit of labour to firms, totalling L
- The initial capital and technology levels are given as $K_0, \ A_0$
- Young individuals divide labour earnings between consumption and savings, carrying over savings into the next period to form the capital for that period
- Old individuals consume all income from past savings plus non-depreciated capital then die
- Firms earn zero profits because of perfect competition with CRS technology – all income is paid to capital and labour

Consumer's Problem

$$\max U = u(c_{1,t}) + \beta\, u(c_{2,t+1})$$
$$\text{s.t. } w_t A_t = c_{1,t} + s_t; \ \ (1 + r_{t+1}) s_t += c_{2,t+1}$$
$$\frac{u'(c_{1,t})}{\beta\, u(c_{2,t+1})} = 1 + r_{t+1}$$

Then use the Euler condition to solve for s_t (if $u(c)$ is known)

Firm's Problem

The firm's profit maximization problem is formulated as:

$$\max_{K_t, L_t^D} F(K_t, A_t L_t^D) - r_t K_t - w_t A_t L_t^D.$$
$$r_t = f'(k_t)$$
$$w_t = f(k_t) - f'(k_t) k_t$$

Market-Clearing Conditions

$$L_t^D = L_t$$
$$K_{t+1} = L_t s_t$$

Substitute r_t, w_t and s_t into capital market to solve for transition equation, giving $k_{t+1} = f(k_t)$, using only exogenous parameters.

Under log-utility and Cobb-Douglas:

$$k^* = \left[\frac{1-\alpha}{(1+n)(1+g)\left(1+\frac{1}{\beta}\right)}\right]^{\frac{1}{1-\alpha}}$$

Stability

The log-utility/Cobb-Douglas version of the model is unique and globally stable, but other versions are not; may have multiple steady-state paths, not all of which are stable.

Social Planner's Problem

Since there are infinite number of generations, the social welfare function is not easy to define. Instead we assess the efficiency of the competitive equilibrium by comparing k^* and $\bar{k}$, the steady-state consumption-maximising level of capital.

Resource constraint:

$$Y_t + K_t = K_{t+1} + L_t c_{1,t} + L_{t-1} c_{2,t}$$

With technology constant and normalized to 1, this becomes:

$$c_1 + \frac{c_2}{1+n} = f(k) - nk$$

Thus, the utility maximising solution is given by:

$$f'(\bar{k}) = r = n$$

Efficiency

It is not necessarily the case that $k^* = \bar{k}$. However, this does not necessarily mean the equilibrium is inefficient.

Under-accumulation ($k^* < \bar{k}$): cannot increase savings without providing a subsidy that would hurt the initial old generation. Hence equilibrium is efficient.

Over-accumulation ($k^* > \bar{k}$): can reduce savings by taxing the young and subsidising the old, making all better off (Pay-as-You-Go social security). Hence equilibrium is not efficient.

Note that a fully-funded social security system does not change the economy's resource constraint, and hence has no effect on the equilibrium outcome.

The Ramsey Model

Setup

- Economy contains L identical, infinitely lived individuals (can normalize L to 1)
- To ensure positive consumption, u satisfies $\lim_{c\to 0} u'(c) = \infty$
- Each individual is endowed with 1 unit of labour in each period, which is supplied inelastically to firms
- The initial capital and technology levels are given as $K_0,\ A_0$
- Firms hire labour and rent capital from individuals to produce output, sell the output in goods market, and pay individuals

- Individuals divide their wealth (labour income, capital income, and remaining savings) into consumption and savings (holding capital), then carry savings to period t + 1

Consumer's Problem

$$\max U = \sum \beta^t u(c_t)$$
$$\text{s.t. } c_t + s_t = w_t A_t + (1+r)s_{t-1}, \quad s_{-1} = k_0$$

Differentiating with respect to s_t means that the problem is simplified to include only c_t and c_{t+1}

$$\frac{u'(c_{1,t})}{\beta\, u(c_{2,t+1})} = 1 + r_{t+1}$$

$$\frac{u'(w_t + r_t s_{t-1} - s_t)}{\beta u'(w_{t+1} + r_{t+1} s_t - s_{t+1})} = 1 + r_{t+1}$$

Consumer's lifetime budget constraint (with taxes):

$$\sum_{t=0}^{\infty} \frac{c_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t - \tau_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{y_t}{(1+r)^t} - \sum_{t=0}^{\infty} \frac{\tau_t}{(1+r)^t}.$$

Government's lifetime budget constraint:

$$\sum_{t=0}^{\infty} \frac{g_t}{(1+r)^t} = \sum_{t=0}^{\infty} \frac{\tau_t}{(1+r)^t}.$$

Firm's Problem

$$r_t = f'(k_t)$$
$$w_t = f(k_t) - f'(k_t)k_t$$

Market-Clearing Conditions

$$L_t^D = L_t$$
$$K_{t+1} = L_t s_t$$

Equilibrium

Substitute the market-clearing conditions, firms problem solutions, and budget constraints all into the Euler equation to solve for the steady-state transition equation. It should be of the form:

$$\frac{u'(A_t f(k^*) - A_t g k^*)}{u'(A_{t+1} f(k^*) - A_{t+1} g k^*)} = \beta[1 + f'(k_{t+1})]$$

Differentiating with respect to k_{t+1} means that the problem is simplified to include only c_t and c_{t+1}

It is not possible to solve for a simple transition equation in the Ramsey model without using the method of undetermined coefficients (i.e. guessing). However, we can easily solve for the steady-state by equating c_t and c_{t+1}.

Social Planner's Problem

Resource constraint:

$$Y_t + K_t = K_{t+1} + L_t c_{1,t} + L_{t-1} c_{2,t}$$
$$f(k_t) = A_t c_t + (1+g)k_{t+1} - k_t$$

Problem:

$$\max U = \sum \beta^t u(c_t)$$
$$\text{s.t.} \quad f(k_t) = A_t c_t + (1+g)k_{t+1} - k_t$$

Which should yield:

$$\frac{u'(c_{1,t})}{\beta u(c_{2,t+1})} = 1 + r_{t+1}$$

This means that the equilibrium allocation is pareto-efficient.

To actually solve this, we take the budget constraints from each period $c_{1t}, c_{2(t+1)}$ and use them to solve for a steady-state value of c. We then take this and substitute it back into the Euler equation. We can solve this for a transition path using the method of undetermined coefficients.

Stability

The lines characterizing c and k can be combined in a phase diagram. For every k, there is a unique level of c that is consistent with the household's optimization and will bring the economy to the steady state. The set of all such combinations of c and k is referred to as the saddle path. Following the saddle path will always lead to economy to a unique steady-state equilibrium point, called the saddle point.

It turns out that the Ramsey model solution is actually consistent with two different trajectories, only one of which corresponds to the saddle path (a unique steady-state solution), which is the trajectory we are interested in.

We can eliminate the extraneous solution by considering the transversality condition, which states that in the limit, either marginal utility or the present discounted value of capital stock (or both) goes to zero:

$$\lim_{t \to \infty} \beta^t u'(c_t) k_{t+1} = 0$$

New Growth Theory

Goals

- Extend on the Solow-model approaches by endogenising A, labour productivity
- Basic idea: sustained growth is driven by sustained growth in technology where the latter is somehow chosen by the agents in the economy
- The main challenge for this literature is to identify what A is and how it evolves over time

The Nature of A

- There are many different types of knowledge, all of which can be embodied in different ways
- There are many channels by which societies accumulate knowledge: formal education, on-the-job training, basic scientific research, learning-by-doing, and innovation
- Knowledge is nonrival and often nonexcludable
- Major forces governing the allocation of resources to the development of knowledge: support for basic scientific research, private incentives for R&D and innovations, alternative opportunities for talented individuals

R&D Based Models

- Arrow Learning by Doing (1962): motivated by the observation that after a new airplane design is introduced, the time required to build the aircraft is inversely related to the number of aircraft of the model that have already been produced; without any evident innovations in the production process. Use this observation to model knowledge as increasing with the stock of capital.
- Romer Spillover Externalities (1986): the accumulation of capital by individuals is associated with a positive externality on the available technology, hence technology proportional to capital per worker
- Romer Research and Competition (1987): agents choose to do research that produces technological improvement, with each innovation representing a technology for producing a new type of intermediate input that can be used in the production of a final good. Continuous expansion of intermediate inputs enables the economy to achieve sustained growth even though each intermediate input taken separately is subject to diminishing returns, and also the welfare losses associated with monopolistic competition.

Analysis of R&D Models

- Virtuous cycles: R&D models generally predict that there will be 'virtuous cycles' of innovation as a result of the positive spillover externalities between firms and countries
- Economic policies: trade, competition, education, taxes and intellectual property all affect the costs and benefits of doing R&D and hence affect the rate of technological progress
- Jones Critique (1995a): Long-run trends in economic growth in the US are not correlated with long-run trends in the various determinants of growth suggested by endogenous growth theories, e.g. R&D intensity, government spending and taxation, subsidies
- Possible counter: Kocherlakota and Yi (1997) find that effects of policies may have 'cancelled out', as they find an increase in public capital tends to raise growth, but the increase in growth is nullified by the increase in taxes needed to finance the expenditure
- Jones Critique (1995b): evidence on productivity growth and R&D inputs in the US and other OECD countries refutes the "scale effect" of R&D models (larger market for a successful innovation, and second by providering a larger stock of potential innovators)
- Worldwide growth: the fact that growth rate of world GDP per capita and growth rate of world population move together is broadly consistent with the theory
- Main problem: R&D models do not explain cross-country growth differences

Human Capital Models

- Augmented Solow Swan: replace L in the model with a new term H, which is a function of L (raw labour hours) and x, the number of years of education/training. Predicts conditional convergence based on population growth, savings rate and years of education
- Lucas Human Capital (1988): representative agent model to predict number of years spent in education. Can generate endogenous growth with sufficiently high returns to education
- Growth Accounting with human capital: efforts to augment labour hours by a quality factor, as determined by years of education or market wages. Some success in explaining cross-country growth differences

Business Cycle Overview

Classical Cycle Approach

- A traditional view of business cycles, summarized by the seminal work of Burns and Mitchell (1946), and has been adopted by the National Bureau of Economic Research
- Business cycles are defined as the recurrent fluctuations in the level of economic activities
- Empirical work on cyclical instability focuses on identifying and dating the phases of the cycle and analyzing the attributes of expansions and contractions
- Using this approach, The NBER has identified 11 business cycles in the US since 1945

Growth Cycle Approach

- Measures fluctuations in real output relative to its long term trend, or the “growth cycle”
- A challenging task of empirical work is to separate the cyclical fluctuations from the long-run growth trend, called “detrending” or “filtering”
- After detrending the data series, researchers typically investigate the features of business cycles from three aspects: amplitude of fluctuations, degree of co-movement with real output, and lead and lag relationships with output

How Detrending Works

- Take logs of time series variable, times by one hundred
- Use band-pass filter, Hodrick-Prescott Filter or similar techniques to separate out trend, cycle, and noise components
- Resulting data shows percentage deviation from trend of relevant variable
- Compare these deviations with the fluctuations in output, and calculate correlations

Summary of Business Cycle Facts

- Employment (total number of employees) fluctuates almost as much as output and aggregate hours, while hours per worker fluctuates much less
- Consumption of nondurables and services fluctuates much less than output, evidence for consumption smoothing
- Investment in both producers’ and consumers durables fluctuates much more than output
- Investment and consumption are both procyclical, with investment slightly moreso
- Trade balance is countercyclical
- Aggregate hours of work are much more procyclical than average labour productivity
- Government purchases are essentially uncorrelated with output
- Employment lags the cycle
- No evidence that monetary base (a variable under the short-term control of the central bank) leads the cycle, i.e., there is little evidence for “money causes output”

Business Cycle Theories

- Classical Theories: focused on empirical observations of fluctuations in output, treated as a sequence of damped oscillations caused by an initial shock to output
- Keynesian Theory: fluctuations caused by changes in aggregate demand, focused on constructing systems of equations of aggregate variables that determined aggregate output, assumed nominal price and wage stickiness

- Real Business Cycles: assumes price flexibility and focuses on the role of productivity shocks in causing fluctuations, models explicitly model consumer and firm behaviour as rational responses to exogenous shocks to build DSGE models
- New Keynesian Theory: adopts the DGSE framework while retaining price stickiness and shocks to AD as main mover of output, stickiness based on microfoundations

Simple RBC Model

Setup

- Simplified RBC model is actually a stochastic version of the Ramsey model
- Question: How much of the cyclical fluctuations in real output is due to productivity shocks?
- Economy is populated with a large number (N) of identical households who live forever
- Each household has an endowment of 1 unit of time in each period, which is divided between work and leisure, denote h_t as the work time
- In addition, the households own an initial stock of capital, K_0 which they rent to firms and may augment through investment
- Individuals observe the production shock z before earning their income and then deciding how much to save and how much to consume. In making this decision, however, they need to predict z_{t+1}

Production Setup

There are a large number of identical firms who have access to a CRS technology, which combines labour (H_t) and capital (K_t) from households to produce the single good (Y_t).

$$Y_t = z_t K_t^{\alpha}(A_t H_t)^{1-\alpha}$$

Where:

$$\begin{aligned} H_t &= \text{labour supplied} \\ A_t &= (1+g)^t \ g \text{ being tech growth} \\ z_t &= \text{stochastic productivity shock} \end{aligned}$$

The productivity shock is assumed to evolve according to:

$$\log(z_t) = \rho \log(z_{t-1}) + \epsilon_t$$

Where: $0 < \rho < 1, \epsilon_t \sim N(o, \sigma^2)$

Steps for Solving the Model

1. Find conditions characterizing the equilibrium by solving social planners problem
2. Find the deterministic steady state by equating $g = 1$, $z = 1$, $\epsilon = 0$ and simplifying
3. Linearize the resulting conditions (resource constraint, Euler, and law of motion for z_t)
4. Solve the linearized equations for policy functions using the method of undetermined coefficients (discard the solution(s) that violates the TVC)
5. Substitute in the unknown variables using the process of calibration
6. Simulate the policy functions using random shocks to find impulse response functions, which give the response over time of endogenous variables to a given-sized external shock

Step 1: Solving for Equilibrium

It is easier to solve the social planner's problem for the equilibrium. We know that it will be the same as the competitive equilibrium as there are no market failures or other distortions in the model.

$$\max U = E_0 \sum \beta^t u(c_t, 1 - h_t)$$
$$\text{s.t.} \;\; c_t + k_{t+1} = z_t (1+g)^{(1-\alpha)t} k_t^\alpha + (1-\delta) k_t$$

To fully solve for the equilibrium we need to find:

- The resource constraint (already given)
- The FOC with respect to k_{t+1}
- The law of motion for productivity shocks (already given)
- Any initial conditions and the transversality condition (already given)

In this model the TVC is given by:

$$\lim_{t \to \infty} E\left(\beta^t u(c_t) k_{t+1}\right) = 0$$

Step 2: Find the Steady State

Step 3: Calibration

Once we have found the equilibrium conditions in terms of parameters, we need to find values for these parameters so we can actually simulate the model. This can be done using various econometrics methods, or it can be done using calibration.

Essentially this means that we select values for exogenous parameters $\alpha, g, \delta, \beta, \rho, \sigma^2$ that are consistent with actual empirical observations of the economy.

α, g and δ are measured directly, so are simply put in the model

β can be estimated by taking the FOC of the social planner's problem, subbing in $(1+g)$ for the ratio of consumptions and $\left(\frac{y}{k}\right)$ for the ratio of capital and technology, and then simply substituting in $\frac{y}{k}$, α, g, δ to solve for β

ρ and σ^2 can be found by writing $\log(z_t)$ as a function of log deviations of output from its 'predicted value' (based on g, H and K), using this equation to estimate a z series based on historical data, and then regressing $\log(z_t)$ on $\log(z_{t-1})$ to directly solve for the coefficient ρ and variance σ^2.

Step 4: Log-Linearization

Log-linearization is a technique that can be used to simplify any non-linear stochastic equation. We often use it in macro to simplify transition equations or equilibrium conditions.

Suppose we have the equation:

$$Y_t = K_t^\alpha H_t^{1-\alpha}$$

We can define a new version of each variable being equal to the log of that variable's deviation from its mean/steady-state value:

$$\hat{Y}_t = \log(Y_t) - \log \bar{Y}$$
$$\hat{K}_t = \log(K_t) - \log \bar{K}$$
$$\hat{H}_t = \log(H_t) - \log \bar{H}$$

These can be re-written in terms of the original variables:

$$\begin{aligned}\widehat{Y_t} &= \log(Y_t) - \log \bar{Y} \\ \log(Y_t) &= \widehat{Y_t} + \log \bar{Y} \\ Y_t &= e^{\widehat{Y_t} + \log \bar{Y}} \\ Y_t &= \bar{Y} e^{\widehat{Y_t}}\end{aligned}$$

$$\begin{aligned}K_t &= \bar{K} e^{\widehat{K_t}} \\ H_t &= \bar{H} e^{\widehat{H_t}}\end{aligned}$$

Substituting these into the original equation and re-arranging we find:

$$\begin{aligned}Y_t &= K_t^{\alpha} H_t^{1-\alpha} \\ \bar{Y} e^{\widehat{Y_t}} &= \left(\bar{K} e^{\widehat{K_t}}\right)^{\alpha} \left(\bar{H} e^{\widehat{H_t}}\right)^{1-\alpha} \\ \bar{Y} e^{\widehat{Y_t}} &= \bar{K}^{\alpha} e^{\alpha \widehat{K_t}} \bar{H}^{(1-\alpha)} e^{(1-\alpha)\widehat{H_t}} \\ \bar{Y} e^{\widehat{Y_t}} &= \bar{K}^{\alpha} \bar{H}^{(1-\alpha)} e^{\alpha \widehat{K_t}} e^{(1-\alpha)\widehat{H_t}} \\ \bar{Y} e^{\widehat{Y_t}} &= \bar{Y} e^{\alpha \widehat{K_t}} e^{(1-\alpha)\widehat{H_t}} \\ e^{\widehat{Y_t}} &= e^{\alpha \widehat{K_t}} e^{(1-\alpha)\widehat{H_t}} \\ e^{\widehat{Y_t}} &= e^{\alpha \widehat{K_t} + (1-\alpha)\widehat{H_t}}\end{aligned}$$

Using the Taylor-series approximation $e^x \approx 1 + x$, we can write this as:

$$\begin{aligned}1 + \widehat{Y_t} &\approx 1 + \alpha \widehat{K}_t + (1-\alpha)\widehat{H}_t \\ \widehat{Y_t} &\approx \alpha \widehat{K}_t + (1-\alpha)\widehat{H}_t\end{aligned}$$

Hence we have re-written the original non-linear equation as a linear equation in terms of log-deviations of the variables from their mean or steady-state values.

Step 5: Solve for Policy Functions

Once we have the linearized equilibrium equations in terms of calibrated parameters, we use the method of undetermined coefficients to solve for a reduced-form analytical solution of the capital, savings and output transition equations.

Step 6: Simulating the Model

Now that the model is fully characterised, we can use a computer program to simulate the response of k_t over time to a one-off z shock of known size. Once the path of k_t is simulated, we can use the equilibrium condition equations to solve for the paths of c_t, y_t and i_t. The resulting paths are called the impulse response functions.

Summary of Findings

- An increase in A (positive z shock) directly increases output and also increases the marginal product of capital such that investment increases to accumulate more capital, which further increases output (transmission mechanism)
- However, the propagation through capital accumulation is fairly small (little amplification of initial shock)
- That means, to account for the cyclical fluctuations in real output by productivity shocks alone, we need very large and persistent productivity shocks, which is not very plausible

- Also, although the simple RBC model replicates the sharp peak and gradual decline to baseline of variables found in the real data, the peak for output occurs too quickly in the model – in reality output takes longer to reach a peak following a positive shock

Comparison of Moments

A more formal way to compare the RBC model to empirical data is to obtain business cycle moments from the model economy and compare them to those observed in the real world. We can then do a comparison and test for statistically significant differences. To do this we generate an artificial dataset from the model by simply using a random number generator to produce a random series of shocks with the appropriate mean and variance, and then computing the moments of this data.

Using this method, we find that the simple RBC model accurately simulates the greater volatility of investment and smaller volatility of consumption compared to output. Both consumption and investment are also procyclical in the model. The main failure of the RBC model is its failure to accurately replicate the dynamics of the labour market (e.g. unemployment), which only fluctuates about half as much in the model as in the real economy. Also, in the model average labour productivity and hours worked move together, whereas in the data they do not.

Extensions to the Model

- Labour market: incorporate search costs, sticky wages, indivisible labour
- Amplification mechanisms: find ways to amplify initial productivity shocks, like variable capital utilization or variable worker effort
- Alternatives to productivity shocks: oil shocks, fiscal shocks (e.g. tax changes), monetary
- Market distortions: efficiency wages, labour hoarding, imperfect competition, financial market frictions, credit constraints
- Other: asset markets and the open economy

The OLG Model and Fiscal Policy

Ricardian Equivalence

- Ricardian equivalence is an economic theory holding that consumers internalize the government's budget constraint: as a result, the timing of any tax change does not affect their change in spending
- If this is true, then government tax cuts in a time of recession would not increase aggregate demand, as households would save the full value of the tax cuts, knowing the debt would eventually have to be repaid
- It would also imply that government budget deficits would not have any ‘crowding out’ effect on private investment, as private savings would always increase one-to-one with public deficits
- Empirical evidence from the post-war US seems to indicate that Ricardian equivalence does not fully hold
- However, empirical findings cannot distinguish between consumers being truly non-Ricardian, and consumers being Ricardian but characteristics of enacted policies do not conform to assumptions under which Ricardian equivalence holds (including lump-sum taxes and knowledge of future incomes)
- The possible outcomes can be investigated using an overlapping-generations model

Model Setup

- The economy has a constant number N of two-period lived individuals
- Each individual is endowed with y_1 goods when young and y_2 goods when old-there is no production
- Individuals can save either through holding capital or government bonds
- The one-period real rate of return to capital is 1 + r, a constant
- Each period, all young individuals pay a lump sum tax τ_1 and old individuals pay τ_2 to the government
- The government has an expenditure of G goods each period, or g per young person

Budget Constraints

Individual:

$$c_{1,t} + \frac{c_{2,t}}{1+r} = y_1 - \tau_{1,t} + \frac{y_2 - \tau_{2,t+1}}{1+r}$$

Government (where b is bond issuances per young person)

$$g_t + (1+r)b_{t-1} = \tau_{1,t} + \tau_{2,t} + b_t$$

Suppose now that there is a tax cut of 100 goods per young person, financed by an equivalent increase in bond purchases.

Case 1: The Crowding Out of Capital

Critical assumption: current young individuals will not have to pay back the debt. Future generations will pay after the government rolls over the debt several times.

Effect on budget constraint:

$$c_{1,t} + \frac{c_{2,t}}{1+r} = y_1 - \left(\tau_{1,t} - 100\right) + \frac{y_2 - \tau_{2,t+1}}{1+r}$$

Thus, reducing the tax by 100 effectively increases lifetime budget constraint by 100 as well. If both c_1 and c_2 are normal goods, both will increase as a result. However for c_2 to increase it is necessary for savings to increase, as:

$$c_2 = y_2 + (1+r)s_t - \tau_{2,t+1}$$

However, for period one consumption to also increase, it must also be true that savings increase by less than the full value of the tax (less than 100), as otherwise consumption in period 1 will remain the same. Thus in this case, savings increase by less than government borrowings increase, so there is a crowding out of private investment.

$$\begin{aligned}
\Delta s_t &= \Delta b_t + \Delta k_t \\
100 - \delta &= 100 + \Delta k_t \\
-\delta &= \Delta k_t
\end{aligned}$$

Case 2: Neutral Government Debt

Critical assumption: Assume now that the debt newly created at t will not be repaid by taxing generations in the future, but by taxing the old at $t+1$.

In order to repay a debt contracted at time t of 100, taxes on the old in $t+1$ must be increased by $100(1+r)$. Thus the consumer's budget constraint becomes:

$$c_{1,t} + \frac{c_{2,t}}{1+r} = y_1 - \tau_{1,t} + \frac{y_2 - \tau_{2,t+1}}{1+r}$$
$$c_{1,t} + \frac{c_{2,t}}{1+r} = y_1 - (\tau_{1,t} - 100) + \frac{y_2 - \left(\tau_{2,t+1} + 100(1+r)\right)}{1+r}$$
$$c_{1,t} + \frac{c_{2,t}}{1+r} = y_1 - \tau_{1,t} + 100 + \frac{y_2 - \tau_{2,t+1} - 100(1+r)}{1+r}$$
$$c_{1,t} + \frac{c_{2,t}}{1+r} = y_1 - \tau_{1,t} + 100 + \frac{y_2 - \tau_{2,t+1}}{1+r} - 100$$
$$c_{1,t} + \frac{c_{2,t}}{1+r} = y_1 - \tau_{1,t} + \frac{y_2 - \tau_{2,t+1}}{1+r}$$

Hence the consumer's budget constraint is completely unaffected by the tax, which means there will be no change in consumption and no change in savings.

The OLG Model and Monetary Policy

Setup

- The economy is populated with 2-period lived overlapping generations. In period t, L_t individuals are born. Population grows at rate n
- There is only one good in the economy, and it cannot be stored between periods
- Each individual receives an endowment of y units of the good when young, and receives no endowment when old
- Consumption in period two is possible by acquiring fiat money in period 1, and exchanging it for goods in period 2
- Define m_t as the units of fiat money acquired per young person, and p_t as the price of unit unit of goods in terms of fiat money

Consumer's Problem

The consumer's budget constraint is given by:

$$c_{1,t} + \frac{m_t}{p_t} = y$$
$$c_{2,t+1} = m_t p_{t+1}$$

$$y = c_{1,t} + \frac{m_t}{p_t}$$
$$y = c_{1,t} + \frac{\frac{c_{2,t+1}}{p_{t+1}}}{p_t}$$
$$y = c_{1,t} + \frac{p_t}{p_{t+1}} c_{2,t+1}$$

$\max U(c_1, c_2)$ s.t. $y = c_{1,t} + \frac{p_t}{p_{t+1}} c_{2,t+1}$

Solve using the Lagrangian to find the Euler equation

Money Market Clearing

This condition is given by:

$$M_t = L_t m_t$$

Re-written in terms of goods:

$$\frac{M_t}{p_t} = \frac{L_t m_t}{p_t}$$
$$\frac{M_t}{p_t} = \frac{L_t(y_t - c_{1,t})p_t}{p_t}$$
$$\frac{M_t}{p_t} = L_t(y_t - c_{1,t})$$
$$p_t = \frac{M_t}{L_t(y_t - c_{1,t})}$$

$$p_{t+1} = \frac{M_{t+1}}{L_{t+1}(y_{t+1} - c_{1,t+1})}$$
$$p_{t+1} = \frac{(1+z)M_t}{(1+n)L_t(y - c_{1,t+1})}$$

This gives the real rate of return of money:

$$\frac{p_t}{p_{t+1}} = \frac{\frac{M_t}{L_t(y_t - c_{1,t})}}{\frac{(1+z)M_t}{(1+n)L_t(y - c_{1,t+1})}}$$
$$\frac{p_t}{p_{t+1}} = \frac{M_t(1+n)L_t(y - c_{1,t+1})}{(1+z)M_t L_t(y - c_{1,t})}$$
$$\frac{p_t}{p_{t+1}} = \frac{(1+n)(y - c_{1,t+1})}{(1+z)(y - c_{1,t})}$$

At stationary equilibrium where consumption plans are the same in all periods:

$$\frac{p_t}{p_{t+1}} = \frac{(1+n)}{(1+z)}$$

Government Budget Constraint

In the case where the government provides a lump sum subsidy to all old citizens funded from the earnings from inflation:

$$L_{t-1}a_t = \frac{M_t - M_{t-1}}{p_t}$$

Equilibrium

This is uniquely defined by the Euler equation, the real rate of return on money, and the government's budget constraint.

Efficiency

This can be checked by checking the golden-rule condition: maximising utility subject to the economy's resource constraint. Note that the final budget conditions should turn out to be the same in equilibrium, however for the maximising problem itself they will be different in equilibrium and golden-rule cases (assuming the equilibrium is inefficient).

To check for efficiency we need to check the welfare of all future generations, and also the welfare of the initial old generation.

Taxation and Efficiency

- Inflation-financed taxation is inefficient because it acts like a proportional tax on the savings of the young
- This causes the young to economise on their savings, thus doing less consumption smoothing than they would like
- A lump-sum tax avoids this disincentive problem and so is pareto efficient
- This means that money is not super-neutral: changes in the money supply do have an effect on the economy, even though the absolute level of prices themselves do not have an effect

The Optimal Rate of Inflation

- Inflation may not be Pareto efficient, but neither are most real-world taxes, and seignorage from inflation is a very cheap way to raise government revenue
- A small amount of inflation is often viewed as having a positive effect on the economy:
    - It is difficult to renegotiate some prices, and particularly wages, downwards, so that with generally increasing prices it is easier for relative prices to adjust
    - Inflation provides an incentive for those with savings to invest them
    - A positive inflation rate also gives central banks room to stimulate the economy
- Another benefit of inflation is that it acts as a tax on the underground economy (which uses cash heavily), and on foreigners holding cash
- However, inflation rates of more than a few percent in industrialised countries seem to be harmful to growth

Exam Tips

- Write conditions (e.g. firm behaviour) in terms of effective labour
- Find GR by starting with overall capital market clearing condition then dividing both by units of effective labour, not forgetting about population and technology growth factors
- Remember universal economy resource constraint Y = C + S
- To 'solve for' the consumption or savings of consumers in any given period means to write them as an equation in terms of any parameters that the consumer takes as exogenous, which means anything except consumptions or savings
- In the OLG model, government taxation will generally be distortionary unless the same individuals have to pay back the debt when they are old. In the Ramsey model it should never make a difference.
- Inada conditions = asymptotically Cobb-Douglas
- The real rate of return on fiat money depends only on the relative growth rates of money (z) and output (n + g); it does not depend on preferences
- The value of money, however, depends upon the ratio of output to goods, and so does depend on preferences, as these will effect capital stocks
- Need to know TVC: $\lim_{t\to\infty} \beta^t u'(c_t) k_{t+1} = 0$
- Generic budget constraint vs generic resource constraint (tax, depreciation, pop growth)
- Calibration: α, g and δ (directly), β FOC, ρ, σ regression
- Money-market clearing condition: $M_t = L_t m_t$
- Ramsey: must sub in k_{t+2}, rearrange, then sub in k_{t+1}
- Differentiate with respect to the correct variable for Lagrangian
- An economy cannot treat its capital stock as income, but an individual can
- RBC: remember to index forward the shock term as well
- Log-linearization: the idea is to sub in the log-deviation form, then use the approximation to get rid of the resulting exponential, and simplify
- Differentiate with respect to the relevant choice variable, in the OLG model generally s, b or m
- Diamond: Euler w.r.t. s, sub in c1 and c2, rearrange to find s, sub in wages to find transition
- Ramsey: Euler w.r.t. k_{t+1}, sub in c1 and c2, sub in w, r and s for k terms, re-arrange for transition equation
- For real rate of return to money, get ratio in terms of m
- Super careful with time subscripts!