Microeconomics

Introduction to Concepts

Deriving the Demand Function

- In order to derive an individual's demand curve/function, we need to know an individual's preferences and his budget constraint

Preferences

- If preferences over bundles of goods satisfy the following properties:
  - completeness (any two bundles can be compared)
  - transitivity (if bundle A is at least as good as B and B is at least as good as C, then A is at least as good as C)
  - continuity (no jumps)
  - monotonicity (always goes up with more goods)
- Then there is a continuous utility function that represent these preferences
- Note that it is possible to satisfy one without satisfying others (e.g. can be complete but intransitive, and vice-versa)
- Some preferences cannot be represented by a continuous utility function; these are referred to as lexicographic preferences, and could occur if a consumer strictly considered the amount of property one before even considering the amount of property two – this could lead to discontinuous preference functions

Strict Monotonicity

- If bundle A contains at least as much of every commodity as the bundle B, then A is at least as good as B
- If bundle A contains strictly more of every commodity than B, then A is strictly better than B
- Due to the strict monotonicity of the preferences the optimal bundle will be on the budget constraint (rather than inside the budget set) - all income will be spent

Convex Preferences

- Strictly convex preferences are present when the average of any two bundles provides greater utility than either of those bundles by themselves
- Preferences are said to be weakly convex if this averaging condition only holds along certain sections of the utility function (e.g. if there are linear segments)
- Convex preferences can be interpreted as preferences for diversity: one peach and one apple is preferred to only two peaches or two apples
- Preferences are said to be strictly convex if they are convex and the indifference curves do not have linear segments; otherwise they are merely weakly convex

Utility Function

- If preferences are represented by a utility function , this utility function is not unique – any positive monotonic transformation of the utility function will represent the same preferences
- We do, however, usually assume that consumer preferences exhibit an additional property of diminishing marginal utility
- This introduces an additional constraint on the potential utility functions that can be used – $q_1 \times q_2$ is a good one to use

Utility Maximisation Problem

- If we combine the budget constraint with an individual's utility function, we can arrive at a constrained utility maximization problem
- We assume here that income and prices are given exogenously, and all the consumer has to decide is what relative quantities to chose in order to maximise the function $U(q_1, q_2)$
- The solution to the utility maximisation problem are the Marshallian (or uncompensated) demand functions q1(p1,p2,Y) and q2(p1,p2,Y)

Solving for the Optimal Bundle

The Optimal Bundle

- The utility maximisation problem is solved at the optimal bundle, which will occur when the indifference curve is tangent to the budget line
- If preferences are strictly convex, the optimal bundle is unique
- The optimal bundle will be set given prices and income, so it is represented $q_1(p_1, p_2, Y)$
- Solving for the optimal bundle thus gives as equations for q1 and q2 given prices and income

Marginal Rate of Substitution

The marginal rate of substitution is the maximum amount of one good that the consumer will be willing to give up in order to obtain one more unit of the other good, at that particular point on the indifference curve (hence the term 'marginal').

The optimal bundle will occur at the point where MRS equals the slope of the budget constraint. This can be derived as follows:

$$\begin{aligned} MRS = dU &= \frac{\partial U}{\partial q_1} dq_1 + \frac{\partial U}{\partial q_2} dq_2 \\ &= MU_1 dq_1 + MU_2 dq_2 \\ \frac{dU}{dq_1} &= \frac{MU_1 dq_1}{dq_1} + \frac{MU_2 dq_2}{dq_1} \\ &= MU_1 + MU_2 \frac{dq_2}{dq_1} \end{aligned}$$

At any point on the indifference curve, $dU/dq_1 = 0$, therefore:

$$0 = MU_1 + MU_2 \frac{dq_2}{dq_1}$$

$$-\frac{MU_1}{MU_2} = \frac{dq_2}{dq_1}$$

The budget constraint is given by:

$$Y = q_1p_1 + q_2p_2$$
$$Y - q_1p_1 = q_2p_2$$
$$q_2 = \frac{Y - q_1p_1}{p_2}$$
$$= \frac{Y}{p_2} - \frac{p_1}{p_2}q_1$$

The slope of the budget constraint is therefore equal to dq_2/dq_1, which as shown above is equal to the MRS. Thus:

$$\frac{dq_2}{dq_1} = -\frac{p_1}{p_2} = -\frac{MU_1}{MU_2} = \frac{dq_2}{dq_1}$$

Solving for the Optimal Bundle

1. Graphically – most useful for perfect substitutes and complements
2. Substitution – using the equation $p_1q_1 + p_2q_2 = Y$, we can rewrite q2 in terms of q1, and then substitute this into the equation max: $U(q_1, q_2)$
3. Lagrange Method – this is useful as it can be used when there are more than two goods

Lagrange's Method

The Langrange function is: $\max \mathcal{L}(x, y, \lambda) = g(x, y) + \lambda f(x, y)$, where λ is the Lagrange multiplier. It is an algorithm that allows us to maximise the function $g(x, y)$ subject to the constraint $\lambda f(x, y)$. Applied to the utility function, the formula becomes:

$$\max U(q_1, q_2) \quad \text{s.t.} \;\; p_1q_1 + p_2q_2 \leq U$$
$$\max \mathcal{L} = U(q_1, q_2) + \lambda(Y - q_1p_1 - q_2p_2)$$

This can be solved by solving the following three equations:

$$\frac{\partial L}{\partial q_1} = 0$$
$$\frac{\partial L}{\partial q_2} = 0$$
$$\frac{\partial L}{\partial \lambda} = 0$$

Economic meaning of the Lagrange multiplier: it equals the marginal utility of each good divided by its price, i.e. the extra utility the consumer will obtain if they are given another dollar in income.

Our goal is to produce the Marshallian demand functions for both goods, written as:

$$q_1 = f(p_1, p_2, Y)$$
$$q_2 = f(p_1, p_2, Y)$$

Where Y is constant

When Lagrange Fails

- If the goods are either perfect complements or perfect substitutes, in which case the Lagrange will give no solution
- If indifference curves are strictly concave to the origin, then the Lagrange will give a minimum rather than a maximum, which will actually be at a corner solution
- For some utility sets and budget lines, MRS=MRT only below the axis. Since negative consumption is impossible, this will actually be a corner solution

Expenditure Minimization Problem

Instead of asking for the maximum utility that can be obtained with a given amount of money, the expenditure minimization problem asks the opposite: what is the minimum amount of money the consumer must spend to achieve at least a given level of utility u if prices are p1 and p2.

The best bundle is where indifference curve is tangent to the isoexpenditure line. The isoexpenditure lines are basically the same as the budget line, except that there are many of them, and the consumer is not restricted to which one they can be on – they just want the lowest that intersects the given indifference curve.

Like the utility maximisation problem, the expenditure minimisation can be solved using the Lagrangian method, as shown below:

$$\min p_1 q_1 + p_2 q_2 \quad \text{s.t.} \; U(q_1, q_2) \geq U$$
$$\min \mathcal{L} = p_1 q_1 + p_2 q_2 + \lambda\big(U - U(q_1, q_2)\big)$$

This can be solved by solving the following three equations:

$$\frac{\partial L}{\partial q_1} = 0$$
$$\frac{\partial L}{\partial q_2} = 0$$
$$\frac{\partial L}{\partial \lambda} = 0$$

Here our goal is to produce the Hicksian demand functions for both goods, written as:

$$q_1^h = f(p_1, p_2, U)$$
$$q_2^h = f(p_1, p_2, U)$$

Where U is constant

Income, Substitution and Welfare

Types of Goods

Normal: A good q_1 is normal if quantity purchased increases with income, holding prices constant

$$\frac{\partial q_1(p_1, p_2, Y)}{\partial Y} \geq 0$$

Inferior: A good q_1 is inferior if quantity purchased decreases with income, holding prices constant

$$\frac{\partial q_1(p_1, p_2, Y)}{\partial Y} < 0$$

Ordinary: A good q_1 is ordinary if quantity purchased decreases with price

$$\frac{\partial q_1(p_1, p_2, Y)}{\partial p_1} \leq 0$$

Giffen: A good q_1 is giffen if quantity purchased increases with price

$$\frac{\partial q_1(p_1, p_2, Y)}{\partial p_1} > 0$$

Note that Giffen goods have to be inferior. As price rise leaving you poorer you may want to substitute towards it from more expensive goods.

Substitute: Good 1 is a substitute for good 2 if the demand for good 1 increases as p_2 rises

$$\frac{\partial q_1(p_1, p_2, Y)}{\partial p_2} > 0$$

Complement: Good 1 is a complement for good 2 if the demand for good 1 decreases as p_2 rises

$$\frac{\partial q_1(p_1, p_2, Y)}{\partial p_2} < 0$$

Indirect Utility Function

The indirect utility function is just the utility function written in terms of prices and income instead of in terms of quantities. It can be found by substituting the Marshallian demand functions into the original utility function.

$$v(p_1, p_2, Y) = U\big(q_1(p_1, p_2, Y), q_2(p_1, p_2, Y)\big)$$

The indirect utility function is homogenous of degree zero in all prices and income:

$$v(tp_1, tp_2 tY) = v(p_1, p_2 Y) \qquad t > 0$$

Thus, if all prices and income are increased by the same proportion at the same time, the utility of the consumer buying the optimal bundle will stay the same.

The Expenditure Function

The consumer's expenditure when they choose optimal bundle for given prices and level of utility is called the expenditure function:

$$e(p_1, p_2, U) = p_1 q_1^h(p_1, p_2, U) + p_2 q_2^h(p_1, p_2, U)$$

The indirect utility function is homogenous of degree one in all prices:

$$e(tp_1, tp_2 tY) = te(p_1, p_2 Y) \qquad t > 0$$

Thus, if all prices are increased by the factor t at the same time, the consumer will have to spend t times as much to achieve utility.

Income and Substitution Effects

If a price of a good increases, it affects the demand for the good through two channels:

- A substitution effect: the change in the quantity of a good demanded when price changes, holding other prices and the consumer's utility constant.
- An income effect: the change in the quantity of a good demanded due to the change in income, holding prices constant.

To decompose these two effects graphically, we can draw a new budget curve at the new prices, but sufficiently high to achieve the same utility as before. The optimal point on this budget line can then be used to judge the size of the substitution effect, with the remainder of the difference between start and end consumption being due to the income effect.

The Slutsky Equation

The skutsky equation simply states that the total effect = substitution effect + income effect.

$$\frac{\partial q_1}{\partial p_1} = \frac{\partial q_1^h}{\partial p_1} - q_1 \frac{\partial q_1}{\partial Y}$$

Note that the first term here $\left(\frac{\partial q_1}{\partial p_1}\right)$ is the inverse of the slope of the Marshallian, or uncompensated demand function (demand holding income constant). The second term $\left(\frac{\partial q_1^h}{\partial p_1}\right)$ is the inverse of the slope of the Hicksian, or compensated demand function (demand holding utility constant).

Measuring Welfare Change

Often we want to know how consumers' welfare is affected by specific events and policies (such as a price increase). Unfortunately we cannot simply observe their utility, so we have to come up with more indirect methods.

One way to measure the decrease in welfare is to ask: How much money do I have to give to the consumer to compensate them for the price increase? If I give the consumer enough money to buy the initial bundle, I overcompensate them, because by substituting towards the now relatively cheaper good, they will actually be able to achieve a higher utility them before. We have to adjust for this when deciding how much to compensate.

One such measure is compensating variation; the amount of money necessary for the consumer to attain the same utility under the new prices:

$$\mathrm{CV} = e(p_1^1, p_2, U_0) - e(p_1^0, p_2, U_0)$$

Another such measure is equivalent variation; the amount of money you have to take from the consumer to harm him/her by as much as the price increase:

$$\mathrm{EV} = e(p_1^1, p_2, U_1) - e(p_1^0, p_2, U_1)$$

In this analysis, we have two Hicksian demand curves: one (CV) corresponds to the original utility, the other (EV) corresponds to the new utility. Both CV and EV measure the area to the left of their respective Hicksian demand curves. Because this area is plotted on a price v. quantity graph, this area thus represents a quantity of money.

In practise, we usually use consumer surplus, which is the area under the Marshallian demand curve (observable), not area under either of the Hicksian demand curves (CV or EV, both of which are unobservable), to measure welfare change. This is a good approximation if the income effect is small.

Producer Theory

The Short Run Production Function

- Capital is fixed: $K = \overline{K}$
- Short run production function: $q = f(\overline{K}, L)$
- Average product of labour: $AP_L = \frac{q}{L} = \frac{f(\overline{K},L)}{L}$
- Marginal product of labour: $MP_L = \frac{\partial q}{\partial L} = \frac{\partial f(\overline{K},L)}{\partial L}$
- Diminishing marginal returns: $\frac{\partial MP_L}{\partial L} < 0$

Returns to Scale

What happens if we increase all inputs by factor t (t>1)?

- Constant returns to scale: $f(tK, tL) = tf(K, L)$
- Increasing returns to scale: $f(tK, tL) > tf(K, L)$
- Decreasing returns to scale: $f(tK, tL) < tf(K, L)$

Cost Minimization

We assume/consider a company trying to achieve the following objective: produce a given output quantity at minimal cost. Prices for output and input are considered to be given exogenously.

Objective: minimise costs subject to $\bar{q}$ (fixed output):

$$\min rK + wL \;\; s.t. \; \bar{q}(K, L)$$

This can be solved using the Lagrangean:

$$\mathcal{L} = rK + wL + \lambda\big(\bar{q} - f(K, L)\big)$$
$$\frac{\partial \mathcal{L}}{\partial K} = 0$$
$$\frac{\partial \mathcal{L}}{\partial L} = 0$$
$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0$$

This yields **conditional factor demands**:

$$K(w, r, \bar{q}) \text{ and } L(w, r, \bar{q})$$

These are homogenous of degree zero in factor prices. Analogous to Hicksian demand function (minimise expenditure given fixed utility).

Plug the conditional factor demands back into the definition of cost $(rK + wL)$ to obtain the **conditional cost function**:

$$C(w, r, \bar{q}) = rK(w, r, \bar{q}) + wL(w, r, \bar{q})$$

This is homogenous of degree one in factor prices.

The method for cost minimisation in the short run is the same, except that capital is fixed: $\bar{K}$.

Long Run Costs

It is always the case that the long run cost curve is the envelope of the short run curves. The two curves will touch when the short run amount of capital is equal to the long run optimal amount of capital. One way of thinking about this is because of the additional constraint on short-run production, it cannot possibly be cheaper than the long-run minimum cost.

Production Technology vs Market Structure

It is useful to retain an analytic separation between these two concepts, as then we can first analyse market structures, and then apply the same rules about cost minimization given production technology to the result, as all firms behave the same in this way.

Production technology: how to produce – cost minimization, the optimal combination of inputs (with smallest cost) to produce a given amount of output

Market structure: how much to produce – profit maximization, the optimal output to produce given the cost function? It depends on the market structure – whether the firm is competitive, a monopoly and so on.

Profit Maximisation

Output is no longer given. We assume that a company chooses inputs and output in order to maximize profits. That is, they take their series of cost-minimising bundles and choose the output level (and hence bundle) that maximises profits.

Objective: maximise profits with respect to K and L:

$$\begin{aligned}\max \pi &= pq - C \\ &= pq(K, L) - rK - wL \\ \frac{\partial \pi}{\partial K} &= 0 \\ \frac{\partial \pi}{\partial L} &= 0\end{aligned}$$

Another approach that yields the same result:

$$\begin{aligned}\max \pi &= pq - C \\ &= pq(K, L) - C(w, r, \bar{q}) \\ \frac{\partial \pi}{\partial K} &= 0\end{aligned}$$

$$\frac{\partial \pi}{\partial L} = 0$$

This yields **unconditional factor demands**:

$$K(w, r, p) \text{ and } L(w, r, p)$$

These are homogenous of degree zero in factor prices. Analogous to Marshallian demand function (maximise utility given fixed income).

Plug the unconditional factor demands back into the production function $f(K, L)$ to obtain the **profit maximising level of output**:

$$q^* = f\big(K(w, r, p), L(w, r, p)\big)$$

The answer will be given in the form of factor demands K(p, r, w) and L(p, r, w), and also the production function q(p, r, w) = f(K(p, r, w), L(p, r, w)).

Shut-Down Decision

Note that in the short run, the firm will only produce the profit maximising quantity q^* if $p \geq AVC(q^*)$; otherwise the firm will be making a loss on each unit of output even excluding fixed costs, and so they will optimally choose to produce zero output. In the long run, the firm will produce q^* if $p \geq AC(q^*)$. We usually assume that firms produce even if they are making zero profit.

This can be represented graphically by plotting the firm's supply curve on a graph of price vs quantity. The competitive firm's SR supply curve is the SR marginal cost curve above the SR average variable cost. Below that the supply curve is always zero.

Partial Market Equilibrium

Partial Equilibrium

- Consider the market for one good (actually two goods: the relevant good and money)
- Assume that prices of all other goods are fixed, regardless of the dynamics of this market
- Given these conditions, derive the equilibrium price for this good
- Consumers: behaviour given by demand function (Consumer Theory)
- Producers: behaviour given by supply function (Producer Theory)

Assumptions of Perfect Competition

- Large number of firms
- Each firm is a price-taker (no strategic interactions)
- Homogeneous product
- Perfect information
- Free entry to and exit from the market

Market Supply

- This is determined by adding each firm's supply curve horizontally
- The result will be a market supply curve that is flatter than all individual supply curves

- Note that each firm produces the optimal SR output: p = MC if price is above or equal to AVC, and zero output otherwise (short run)
- The profit for each firm could be positive, zero, or negative (in the short run)
- In the long run the number of firms is not fixed: Firms will enter if profits are positive, while Firms will exit if profits are negative
- Free entry and exit will thus lead to zero profits in the industry in the long run

Returns to Scale

- The shape/slope of the LR market supply curve is determined by whether the industry is a constant-cost industry or an increasing-cost industry
- In a constant cost industry, the LR market supply curve is always a horizontal line at marginal cost, where each firms earns zero profits
- If price is higher, profits are infinite, firms are willing to produce infinite quantity and new firms want to enter; if price is lower, quantity supplied is zero (profits will be negative)
- It does not matter what the cost curves of each firm look like (e.g. could be increasing marginal cost), as each firm simply produces at p=MC, and then the correct number of firms enter or leave the market to compensate
- This is not the case for the SR industry supply curve, as the only way for SR supply to change is for existing firms to alter levels of production (no entry or exit in SR)

Equilibrium

- At the equilibrium, supply = demand
- The LR equilibrium price is determined by: p = min AC(LR) = MC(q1) – this is where profits are zero
- The industry output will be determined by the quantity demanded at this price
- The number of firms in the industry is qT/q1

Equilibrium in a Monopoly Market

- Monopoly is the single producer of the good, and so it chooses the price p to charge that will maximise profits (MR=MC), and then sells the quantity demanded at this price

$$MR(q) = p(q) + \frac{dp(q)}{dq} q = p(q)\left[1 + \frac{dp}{dq}\frac{q}{p}\right] = p(q)\left[1 + \frac{1}{\frac{dq}{dp}\frac{p}{q}}\right] = p(q)\left[1 - \frac{1}{|\varepsilon(q)|}\right]$$

- Note that if the monopolist's demand curve is linear, then the MR is a linear function with twice the slope as the demand curve

Types of Price Discrimination

- First degree: charge different price for each unit of the good, and sell each unit of the good to the customer with the highest marginal willingness to pay for that unit; must be able to prevent resale and know valuation
- Second degree: nonlinear pricing such that price depends on quantity purchased, produces are bundled, or there are 'two-part' prices
- Third degree: different prices for different groups, based on age, location, time, etc; must be able to prevent resale between groups

A Note on Demand

- When solving problems with more than one type of demand function (different types of consumers), demand functions should be graphed and then added horizontally to get the total demand, just as for total industry supply
- It is also a good idea to solve the problem in general form first (e.g. substitute unknowns for the parameters that differ across the different demand functions), then just sub in particular values for specific cases

Pure Exchange Economy

The Edgeworth Box

- The quantities of two goods are represented on one of the two axes of a box
- Each consumer has an endowment bundle of these two goods
- Consumers also have indifference curves representing bundles of equal utility; one consumer's lines emanate from the bottom left corner, and the other the top-right corner
- Consumers are able to trade, and are assumed to have protected property rights and independent consumption utilities
- Any point in the Edgeworth box represent some possible allocation of total endowment between consumers. Such an allocation is called a feasible allocation

Perfectly Competitive Equilibrium

- Each consumer takes prices of the goods as given; represented as the budget line
- Each consumer sells his/her endowment at the market price (supply) and buys the bundle that maximizes his/her utility (demand) given the prices
- Equilibrium condition (market clearing condition): at the equilibrium prices the demand for each good is equal to the supply of that good
- The budget line must be the same for both consumers (as the face the same prices), but only the 'right' budget line produced by market-clearing prices will correspond to an equilibrium
- Note that goods in an edgeworth box may be inferior (as opposed to normal), so prices could rise or fall at any given point of disequilibrium
- At equilibrium: $MRS_A = MRS_B = \frac{p_1}{p_2}$; this is called the Walrasian Equilibrium
- This means that prices equate MRSs across all consumers, and therefore goods are distributed efficiently

Calculating the Equilibrium

- All we need to know to calculate the equilibrium are initial endowments and the utility functions of each individual – no income, because that is given by endowments and prices

$$U(q_1, q_2) = q_1^{\alpha} q_2^{1-\alpha}$$
$$\max \mathcal{L} = U(q_1, q_2) + \lambda(Y - q_1 p_1 - q_2 p_2)$$

$$\frac{\partial L}{\partial q_1} = 0$$
$$\frac{\partial L}{\partial q_2} = 0$$

$$\frac{\partial L}{\partial \lambda} = 0$$

Solving these out we find the following demand functions:

$$q_1(p_1, p_2) = \frac{\alpha Y}{p_1}$$
$$q_2(p_1, p_2) = (1-\alpha)\, Y/p_2$$

Income is given by endowments, therefore:

$$q_1^A(p_1, p_2) = \frac{\alpha(\omega_1 p_1 + \omega_2 p_2)}{p_1}$$
$$q_2^A(p_1, p_2) = \frac{(1-\alpha)(\omega_1 p_1 + \omega_2 p_2)}{p_2}$$

Knowing that $q_1^A + q_1^B = q_1^T$ and $q_2^A + q_2^B = q_2^T$, we can then solve for the price ratio. Note that we can never solve for actual prices because only relative prices matter: demands are homogenous of degree zero in prices.

Walrus' Law

- Aggregate excess demand for good 1 is: $z_1(p_1, p_2) = q_1^A(p_1, p_2) + q_1^A(p_1, p_2) - \omega_1^A - \omega_1^B$
- Aggregate excess demand for good 2 is: $z_2(p_1, p_2) = q_2^A(p_1, p_2) + q_2^A(p_1, p_2) - \omega_2^A - \omega_2^B$
- At the equilibrium (market clearing condition), aggregate excess demand for each good is zero: $z_1(p_1^*, p_2^*) = 0,\ z_2(p_1^*, p_2^*) = 0$
- If preferences are strictly monotone, Walras' Law holds: $p_1 z_1(p_1, p_2) + p_2 z_2(p_1, p_2) = 0$
- This means that the value of aggregate excess demand is zero (for any prices, even non-equilibrium prices)
- From Walras' law it can be determined that if one market is in equilibrium, then the other is in equilibrium (for two markets)
- Also, if (n -1) markets are in equilibrium, then the last market is in equilibrium (for n markets)

First Welfare Theorem

- A feasible allocation Y Pareto dominates a feasible allocation X if at Y someone is better off than at X and no one is worse off
- A feasible allocation X is Pareto optimal if there is no other feasible allocation that Pareto dominates it
- Any allocation here indifference curves cross is Pareto dominated by any allocation inside the contract curve
- The set of all Pareto optimal points is called the contract curve
- The first welfare theorem states that the Walrasian Equilibrium is Pareto optimal: perfectly competitive markets are efficient because the MRS are the same across all consumers, and hence there are no possibilities for a mutually beneficial exchange are left

Second Welfare Theorem

- Any Pareto optimal allocation can be achieved as a Walrasian Equilibrium

- This means that distributional concerns can be addressed through the market mechanism
- By using lump-sum taxes/ wealth transfers the government could change the initial endowments in such a way that any given PO allocation will be a result of the competitive market mechanism (will be supported as a WE)

General Equilibrium with Production

The Efficient Outcome with PPF

- First we must define a Production Possibility Frontier for each producer making each good in the economy
- It is represented as a production possibility frontier, the slope of which is the marginal rate of technological transformation (or just MRT = marginal rate of transformation)
- It shows how many of good A can be traded off for good B at any particular level of output
- The optimum point of consumption/production occurs where the indifference curve is tangent to the PPF
- At this point, MRTT = MRS, meaning that the tradeoff in utility between the goods is equal to the tradeoff in production between them

Producer's Choice

- The producer's isoprofit line marks out points of equal profit on a graph of output versus labour (or capital) input
- The isoprofit line is given by $q = \frac{w}{p}z + c$, where c is the optional intercept and z is the labour input, w is waged and p is price
- On the same graph we can also plot the production function, which represents output as a function of one input
- Profit is maximized at the point where production function is tangent to the isoprofit line, as this will be the highest isoprofit line the firm can obtain
- At this point, the slope of the production function equals the slope of the isoprofit line: $MP_L = w/p$

Consumer's Choice

- Consumers have preferences given by a utility function, the slope of which represents the MRS (marginal rate of substitution)
- The budget line faced by the consumer is found by converting their endowments to cash at market prices, and then summing the result across all endowment goods
- The optimum consumption point will be where utility is maximised, which occurs at the point where the indifference curve is tangent to the budget line
- At this point, $MRS = w/p$

Competitive Equilibrium

- Market clearing conditions are as follows: demand for coconuts by consumer = supply of coconuts by the producer, and demand for labour from the producer = supply of labour by the consumer

- This is satisfied when the production function is tangent to the isoprofit line, and also when the indifference curve is tangent to the budget curve
- Because the budget curve and the isoprofit line are essentially the same (as consumer and producers face the same prices), it must be the case that MPL = MRS = w/p
- The only way for this to occur when consumers and producers are maximising profits and utility is if there are either surpluses and/or shortages, which would not be an equilibrium
- This equilibrium point is pareto optimal, as both producer and consumer are maximising their outcomes

Two-Producer Economy

- Productive Efficiency
- We can draw the isoquants for each of two firms (over a single input) in an edgeworth box
- The points of efficient production occur where the two isoquants are tangent; at all other points we could produce more of each good by simply re-allocating inputs between producers
- Allocative Efficiency
- We can now take the path of efficient production points from the output edgeworth box and use it as a PPF, with one good on each axis
- We can then draw a two-consumer edgeworth box within this PPF
- An equilibrium price will be established that equalises the MRS for both consumers, and also is equal to the MRT at the relevant point on the PPF

The Role of Prices

- Prices equate MRS between consumption goods among all consumers (trade off in preferences is the same for all consumers)
- Prices equate MRTS between inputs among all producers (trade off in inputs' use is the same for all producers)
- Prices equate MRTT between consumption goods (outputs) to the MRS between goods (trade off in production for the whole economy = trade off in consumption for each consumer)

Three-Fold Optimality

- Allocative efficiency: Goods are distributed efficiently among the consumers (MRS are equal for all consumers)
- If this isn't the case, we can redistribute goods between consumers so someone is better off and no one is worse off
- Production efficiency: Goods are produced efficiently (MRTS are equal for all producers)
- If not, we can redistribute inputs between firms so some firms produce more and none produce less than before
- Output efficiency: Correct amount of goods is produced (MRTT = MRS for any two goods)
- If MRTT is not equal to MRS for some goods, we can change the economy's production point (by redistributing inputs among firms) and change consumption point of consumers in such a way that at least one consumer is better off and no one is worse off

The Benefits of Trade

- International trade permits the optimal production and optimal consumption points to occur at different locations in the PPF diagram
- This is possible because the externally given price ratio permits consumption to occur at a point that was previously outside the PPF
- One nation/individual specialises in producing one good, while the other specialises in producing the other good; they both trade the difference

Choice Under Uncertainty

Probability and Lottery Basics

- Suppose the outcome of some process may take n possible values, then the set of outcomes is Ω={1,...,n}.
- The set p = (p_1, p_2,...., p_n) is called a probability distribution on the set of outcomes if each p is positive and all p's sum to 1
- To specify a lottery we need to specify values x_i of possible outcomes and their probabilities p_i ; this can be represented as (x_1, p_1;...x_i, p_i;...x_n, p_n)
- The expected value of a lottery is defined by $\sum p_i x_i$
- The variance of a lottery is defined by $\sum p_i (x_i - Ex)^2$
- Standard deviation provides us with a crude measure of risk

Expected Utility Theory

- Rather than having preferences over bundles of goods, we now have preferences over the lotteries (risky alternatives)
- All three axioms of normal consumer theory (completeness, transitivity and continuity) still hold, plus an additional one of independence
- The independence assumption states that: Then for any lottery $L \Rightarrow \alpha L_1 + (1\text{-}\alpha)L \succeq \alpha L_2 + (1\text{-}\alpha)L$
- This just essentially means that preferences are independent of context and are not altered by the addition of other choices or complicating factors (e.g. if you prefer A to B then you also prefer A + c to B + c
- If these four axioms are satisfied then preferences over lotteries can be represented by the expected utility function: $EU(L) = p_1 u(x_1) + p_2 u(x_2) + \cdots + p_n u(x_n)$
- This also means that if $L_1 \succcurlyeq L_2$ then $EU(L_1) \geq EU(L_2)$
- The expected utility function is defined up to a positive linear transformation: for the two utility functions U(L) and V(L), they are the same if U(L) = aV(L) + b
- This is much more restrictive than the requirement in certain utility functions, which are all the same if bundles are ordered in the same way up to any positive monotonic transformation

Attitudes Towards Risk

- An individual is risk averse if she prefers the certainty of the expected value of a lottery to the lottery itself: concave (diminishing marginal) Bernoulli utility function
- An individual is called risk neutral if she is indifferent between the expected value of a lottery and the lottery itself: linear Bernoulli utility function

- An individual is called risk lover if she prefers the lottery itself to the certainty of having the expected value of a lottery: convex (increasing marginal) Bernoulli utility function

The Market for Insurance

- Only risk averse individuals will be interested in buying insurance
- Suppose the consumer has wealth w, and with probability q > 0 she may suffer an accident, in which case her wealth will be reduced to w - D. Assume 0 < D < w.
- If she insures for the amount I, she has to pay the insurance premium x = rI, where r < 1 is described in the insurance contract
- The insurance company will repay her I in the case of an accident
- Therefore, her expected utility is: $EU(I) = (1-q)U(w-rI) + qU\big(w - D + (1-r)I\big)$
- Under an actuarially fair policy the insurance company breaks even on average: $\Pi = 0 = x - qI = rI - qI = (r-q)I \therefore r = q$
- To find the optimum amount of insurance purchased, we must take the rest that r = q and substitute it into the derivative of EU(I)
- Doing so, we find that I = D, or in other words, the amount of insurance = the amount of possible loss
- The policy leaves the customer with the same expected wealth, whether or not the accident occurs – this is called full insurance
- A risk-averse individual insures fully if the price of insurance is actuarially fair

Investment in a Risky Asset

- Consider a risky asset that pays z_1 with probability p and z_2 with probability $1-p$ per dollar invested
- Assume z_2 > 1 > z_1, i.e. there is a genuine risk of losing and a genuine chance of winning
- Assume that the asset is actuarially favorable: $pz_1 + (1-p)z_2 > 1$, i.e. the expected (gross) return is higher than the cost (required investment)
- Consider an individual with Bernoulli utility function u(·) who possesses initial wealth w and invests x in risky asset
- Therefore, her expected utility is: $EU(x) = pU(w - x + z_1x) + (1-p)U(w - x + \ z_2x)$
- Once again, we can differentiate this and equate to zero to find the maximum EU
- With this method, it can be shown that an investment of x = 0 is never optimal, regardless of one's risk preference; thus all people will invest a positive amount

Comparing Risk Preferences

- Risk-aversion is captured by concavity of the utility function
- The degree of concavity of a function is captured by the magnitude of its second derivative
- As such, Arrow and Pratt introduced the following absolute risk-aversion coefficient: $r_A(x,u) = -\frac{u''(x)}{u'(x)}$, where the negative sign out the front ensures that the value is positive for risk averse individuals
- A similar measure is the relative risk-aversion coefficient: $r_R(x,u) = -x\frac{u''(x)}{u'(x)}$
- The greater either of these coefficients are, the more risk averse the individual is

The Certainty Equivalent of a Lottery

- The certainty equivalent of a lottery is the amount of certain money that will give you the same utility as the lottery: $EU(L) = U(CE_w)$
- If it is the case that $CE < E(L)$ then the individual is risk averse
- The risk premium of a lottery is equal to the expected value of the lottery minus the certainty equivalent wealth
- It is equal to the amount of money the individual is willing to pay to avoid the gamble (i.e. to get the expected value of the lottery for certain instead of lottery itself)

Game Theory

Defining a Strategic Game

- Requires a set of players: i = 1, 2,…I
- Has a set of strategies for each player: $S_i = \{s_i{'}, s_i{''}, \ldots\}$
- Finally, there is a payoff for each player for each possible strategy profile $(s_1, s_2, \ldots s_N)$ given by $u_i(s_1, s_2, \ldots s_N)$

Equilibrium with Dominant Strategies

- If a player has a strategy that is always the best, no matter what the other player does, then we predict that the player will play this dominant strategy
- The strategy s_i' is a dominant strategy for the player i if it gives the player at least as high payoff as any other of his strategies for any strategies of other players
- Equilibrium in dominant strategies is believable, but it does not exist for most games!
- This is because it does not capture the strategic interaction element of most game theory problems, as each player has a best strategy regardless of what anyone else does

Nash Equilibrium

- The strategy profile $a^* = (a_1^*, a_2^*, \ldots a_N^*)$ is a Nash equilibrium if for every player her strategy a_i^* gives her at least as large a payoff as any other of her strategies, given that all other players play strategies their NE strategies a_{-i}^*
- This is written as: $u_i(a_i^*, a_{-i}^*) \geq u_i(a_i, a * -i)$ for any i and any a_i
- In other words, in NE each player plays their best response strategy to the strategies of other players
- As such, no player has an incentive to unilaterally deviate from their strategy given the strategies of other players

Finding the Nash Equilibrium

- There are two ways to find the Nash Equilibrium:
- Find best response for each player, then find all strategy profiles where best responses intersect (each player plays a best response to strategies of other players)
- All “intersections” are NE, and if it is not an “intersection”, then it is not a NE
- Guess the NE, and then check to see if it is an NE by verifying that no player has an incentive to deviate (everyone is playing their best response)

- A Nash equilibrium can be viewed as a self-reinforcing agreement, or a result of learning from past experience

Mixed Strategies

- The examples we have looked at so far involve pure strategy Nash equilibrium
- In other words, in equilibrium, all players chose one strategy 'for certain'
- A mixed strategy Nash equilibrium exists when each player 'randomises' between strategies
- The 'matching pennies' game is an example of a mixed strategy game
- The NE in such a game will be some probability of playing the pure strategy and not playing the pure strategy of each player

Cournot Oligopoly

- Suppose there are two identical firms (duopoly) producing identical products
- Set of strategies for each player: each firm decides how much output to produce before it goes to sell the product (it can produce any non-negative output q)
- Assume that demand is $p = a - Q$, where Q = $q_1 + q_2$ is the total output, the price of the good adjusts to clear the market, and the marginal cost is **c** (and no fixed costs)
- Therefore, the profit of each firm is:

$$\pi_i = pq_i - cq_i = (a - q_1 - q_2)q_i - cq_i = (a - q_1 - q_2 - c)q_i$$

- To determine the behaviour of firm 1, we must maximise this function with respect to q_1 and then rewrite this equation as a function of q_2:

$$\begin{aligned}\frac{\partial \pi}{\partial q_1}(a - q_1 - q_2 - c)q_1 &= a - 2q_1 - q_2 - c \\ 0 &= a - 2q_1 - q_2 - c \\ 2q_1 &= a - q_2 - c \\ &= \frac{1}{2}(a - c - q_2)\end{aligned}$$

- This is the best response function of the firm 1: for each possible output of the firm 2 it specifies the optimal (profit-maximizing) output of firm 1
- We can calculate the same equation for firm 2, and then substitute this back into the best response function of firm 1 to find the equilibrium:

$$\begin{aligned}q_1 &= \frac{1}{2}\left(a - c - \left[\frac{1}{2}(a - c - q_1)\right]\right) \\ &= \frac{1}{2}\left(a - c - \frac{1}{2}a + \frac{1}{2}c + \frac{1}{2}q_1\right) \\ &= \left(\frac{2}{4}a - \frac{2}{4}c - \frac{1}{4}a + \frac{1}{4}c + \frac{1}{4}q_1\right) \\ q_1 &= \frac{1}{4}a - \frac{1}{4}c + \frac{1}{4}q_1 \\ \frac{3}{4}q_1 &= \frac{1}{4}a - \frac{1}{4}c \\ q_1 &= \frac{4}{12}a - \frac{4}{12}c \\ &= \frac{1}{3}a - \frac{1}{3}c\end{aligned}$$

- Note that the nash equilibrium in a cournot duopoly provides a higher quantity and lower price than the monopoly outcome
- This occurs because the inframarginal loss of each additional unit sold for each firm decreases (because of the additional firm), while marginal gain for unilateral increases in production is the same. Thus, equilibrium output is higher than under the monopoly

Bertrand Oligopoly

- In this model, firms set their price and then decide how much to produce at that price
- Under this assumption, there is the unique Nash equilibrium when both firms charge price equal to marginal cost
- His means that the Bertrand oligopoly result is the same as the PC market
- No firm will charge price smaller than c in NE:
- If p < c for at least one firm, then the firm with lowest price has negative profit and would be better off by charging p = c (and earning zero profit): therefore not NE
- If pi = c and pj > c , then the firm i would be better off by increasing its price just a little bit (and earning positive profit): therefore not NE
- If both prices pi > c and pj > c, then the firm with higher (or equal) price would be better off by decreasing its price to the level just a little bit below the price of the other firm (or to the monopoly price, whichever is smaller). As a result, this firm will capture all consumers and increase its profit: therefore not NE

Extensive Form Games

- The key feature of extensive form games is that there is an explicit specification of the order of actions of the players
- Because one player observes the action of the other player before making their move, we can get very different outcomes than in the case of simultaneous games
- For a strategic form game, a player's strategy is a complete contingent plan of their actions, with her actions specified at each node that belongs to them, even for nodes that will not actually be reached
- An information set is a group of nodes at which the player to whom the nodes belong has the same information about what occurred previously in the game
- A subgame is a portion of the game tree that begins at a node that is a singleton information set (in games with perfect information each node is a singleton information set), and includes all nodes that follow (but no other nodes)
- An equilibrium of the game is subgame perfect (SPNE) if it stays Nash equilibrium in every subgame – that is, it does not rely on any 'incredible threats'
- All SPNE can be found using the method of backward induction

Stackelberg Duopoly

- This is a dynamic game with continuous strategies and two firms that produce identical goods
- Firm 1 is the leader: It decides how much output to produce first q_1
- Firm 2 is the follower: it observes the choice of the firm 1 and then decides how much output q_2 to produce

- The price of the good will adjust to clear the market: $p(Q) = a - Q$, where $Q = q_1 + q_2$ is the total output
- The marginal cost for each firm is **c** (and no fixed costs)

Moral Hazard

The Economics of Information

- The economics of information discusses situations where one of the contracting parties has private information which may affect the utility of the other party; asymmetric information
- The models of these type can be divided into two broad categories:
- Hidden information models (informational asymmetries exist prior to contracting)
- Hidden action models (the information is symmetric prior to contracting, but an asymmetry arises after the contract is signed)

What is Moral Hazard?

- Moral Hazard: after the contract is signed one party can take an action that is unobservable by the other party, but also affects the payoffs of the other party
- When writing the contract the parties should thus anticipate the possibility of an opportunistic behavior by the other party
- So the question is: how to design a contract that mitigates the difficulties caused by the asymmetric information?
- Most moral hazard problems deal with situations when one individual hires the other to take some action for him as his agent

The Principal-Agent Model

- Suppose a principal hires an agent to do some work for her
- She offers the contract to the agent, who either accepts or rejects it
- If agent signs the contract, he then chooses how much effort to exert
- The higher the effort level, the higher is the expected gross profit for the principal and the higher is the disutility of effort for the agent.
- It is also assumed that the principal is risk neutral and the agent is risk averse
- If effort is not observable and cannot be included in the contract, then the contracts that are best under symmetric information (no moral hazard) are no longer adequate

please note that most students find Honours Micro (and other subjects) quite challenging. It might be a good idea to talk to current Honour students about what you are to expect and what you can do now.

If you want to have easier life at Honours, I suggest you learn as much math and Micro as you can now (including some advanced stuff that is not required for ECON30010(316-313)). Please see the Honours Micro [ECON40001(316-402)] outline for more information. *Advanced Micro Theory* by Jehle and Reny is used for this subject - please check it out to see what is coming.

If you are planning to do PhD afterwards, you need even more math - so you might consider taking some classes at math department - see advice for future PhD students.

Graduate Studies in Economics - PhD

Advice on **Mathematical Preparation for future PhD students** (in US):

http://www.vanderbilt.edu/AEA/gradstudents/Mathematical_Preparation.htm

Here is an advice provided by AEA (**American Economic Association**) for students interested in doing **PhD program** in Economics (in US, but Australia and Europe is more or less similar):

http://www.vanderbilt.edu/AEA/gradstudents/index.htm

It provides general information about graduate studies, Universities and so on. If you are interested in an academic career (anywhere in the world), please read it now.

Almost every year, students who graduated from the University of Melbourne are admitted to a top US PhD program. Ask them for advice and information if you can.