Macroeconomics

Economic Growth

Solow-Swan Model

Assumptions

- Firms/economy have production function
- Population increases over time (exogenous),
- There is full employment (pop=L)
- Savings equals investment
- Generally also assume constant returns to scale

Production Function

$$K_t = \text{capital stock time } t$$
$$L_t = \text{labour input at time } t$$
$$F = \text{production function displaying CRS}$$

$$Y_t = F(K_t, L_t)$$
$$\frac{Y_t}{L_t} = y_t$$
$$\frac{Y_t}{L_t} = F\left(\frac{K_t}{L_t}, \frac{L_t}{L_t}\right) = F\left(\frac{K_t}{L_t}\right)$$
$$\therefore y_t = f(k_t)$$

Capital Accumulation

$$\delta = \text{rate of capital depreciation}$$
$$I_t = \text{investment at time } t$$
$$n = \text{growth rate of labour}$$
$$s = \text{savings rate}$$
$$c = \text{consumption per capita}$$

$$K_{t+1} = (1-\delta)K_t + I_t$$
$$I_t = sY_t$$
$$L_{t+1} = (1+n)L_t$$

$$\frac{K_{t+1}}{L_{t+t}} = \frac{(1-\delta)K_t + I_t}{L_{t+t}}$$
$$k_{t+1} = \frac{(1-\delta)K_t + sY_t}{(1+n)L_t}$$
$$= \frac{(1-\delta)K_t}{(1+n)L_t} + \frac{sL_t f(k_t)}{(1+n)L_t}$$
$$= \frac{(1-\delta)k_t}{(1+n)} + \frac{sf(k_t)}{(1+n)}$$
$$k_{t+1} = \frac{(1-\delta)k_t + sf(k_t)}{(1+n)}$$

Finding the Steady State

Steady state occurs when $k_{t+1} = k_t$

$$k^* = \frac{(1-\delta)k^* + sf(k^*)}{(1+n)}$$
$$k^*(1+n) = (1-\delta)k^* + sf(k^*)$$
$$k^*(1+n) - (1-\delta)k^* = sf(k^*)$$
$$k^* + k^*n - k^* + \delta k^* = sf(k^*)$$
$$k^*(n+\delta) = sf(k^*)$$
$$k^* = \frac{sf(k^*)}{(n+\delta)}$$

Assuming Cobb-Douglas production technology then

$$Y_t = AK_t^{\beta} L_t^{1-\beta}$$
$$y_t = \frac{AK_t^{\beta} L_t^{1-\beta}}{N_t}$$
$$= AK_t^{\beta} L_t^{-\beta}$$
$$= \frac{AK_t^{\beta}}{L_t^{\beta}}$$
$$= Ak_t^{\beta}$$

$$\frac{K_{t+1}}{L_{t+t}} = \frac{(1-\delta)K_t + I_t}{L_{t+t}}$$
$$k_{t+1} = \frac{(1-\delta)K_t + sY_t}{(1+n)L_t}$$
$$= \frac{(1-\delta)K_t}{(1+n)L_t} + \frac{sAK_t^{\beta} L_t^{1-\beta}}{(1+n)L_t}$$
$$= \frac{(1-\delta)k_t}{(1+n)} + \frac{sAk_t^{\beta}}{(1+n)}$$
$$k_{t+1} = \frac{(1-\delta)k_t + sAk_t^{\beta}}{(1+n)}$$

$$k_t^* = \frac{sAk_t^{\beta}}{(n+\delta)}$$
$$\frac{k_t^*}{k_t^{\beta}} = \frac{sA}{(n+\delta)}$$
$$k_t^{*1-\beta} = \frac{sA}{(n+\delta)}$$
$$k^* = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\beta}}$$

The Golden Rule

$$c_t = y_t - sy_t$$
$$c_t^* = f(k^*) - sf(k^*)$$

$$c_t^* = f(k^*) - k^*(n+\delta)$$
$$\max c_t^* = \frac{dc_t^*}{dk_t}$$
$$0 = f'(k^*) - (n+\delta)$$
$$f'(k^*) = (n+\delta)$$

Assuming Cobb-Douglas production technology then

$$f'(k^*) = \alpha k^{\alpha-1} = n+\delta$$
$$k^* = \left(\frac{\alpha}{n+\delta}\right)^{\frac{1}{1-\alpha}}$$

$$\frac{k^*(n+\delta)}{f(k^*)} = s^*$$
$$s^* = \frac{k^*(n+\delta)}{k^{*\alpha}}$$
$$= (n+\delta)k^{*1-\alpha}$$
$$= (n+\delta)\left(\left(\frac{\alpha}{n+\delta}\right)^{\frac{1}{1-\alpha}}\right)^{1-\alpha}$$
$$= (n+\delta)\left(\left(\frac{\alpha}{n+\delta}\right)^{\frac{1-\alpha}{1-\alpha}}\right)$$
$$= (n+\delta)\left(\frac{\alpha}{n+\delta}\right)$$
$$s^* = \alpha$$

Technological Progress

$Y_t = F(K_t, E_tL_t)$

E_t = contribution of technology to labour productivity

g = technological growth rate

$E_{t+1} = (1+g)E_t$

$L_{t+1} = (1+n)L_t$

$$\frac{Y_t}{E_tL_t} = \tilde{y}_t$$
$$\frac{K_t}{E_tL_t} = \tilde{k}_t$$

$$\frac{K_{t+1}}{E_{t+1}L_{t+1}} = \frac{(1-\delta)K_t + sY_t}{(1+g)E_t(1+n)L_t}$$
$$= \frac{(1-\delta)K_t}{(1+g)(1+n)E_tL_t} + \frac{sY_t}{(1+g)(1+n)E_tL_t}$$
$$= \frac{(1-\delta)\tilde{k}_t}{(1+g)(1+n)} + \frac{s\tilde{y}_t}{(1+g)(1+n)}$$
$$\tilde{k}_{t+1} = \frac{(1-\delta)\tilde{k}_t + s\tilde{y}_t}{(1+g)(1+n)}$$

Steady state when $\tilde{k}_{t+1} = \tilde{k}_t = \tilde{k}^*$

$$\tilde{k}^*(1+g)(1+n) = (1-\delta)\tilde{k}^* + s\tilde{y}_t$$
$$\tilde{k}^*(1+g)(1+n) - (1-\delta)\tilde{k}^* = s\tilde{y}_t$$
$$\tilde{k}^*[g+n+gn+1-1+\delta] = s\tilde{y}_t$$
$$\tilde{k}^* = \frac{s\tilde{y}_t}{g+n+\delta+gn}$$

Finding the Growth Rate

$$Y_{t+1} = (1\ +\ g)(1\ +\ n)Y_t$$
$$\frac{Y_{t+1}-Y_t}{Y_t} = \frac{\big((1\ +\ g)(1\ +\ n)-1\big)Y_t}{Y_t}$$
$$= (1\ +\ g)(1\ +\ n) - 1$$
$$= g+n+gn+1-1$$
$$\approx g+n$$

Growth Accounting Methods

$$\frac{dY}{dt} = \frac{dF(K_t, E_tL_t)}{dt}$$
$$= \frac{\partial Y}{\partial K}\left(\frac{dK}{dt}\right) + \frac{\partial Y}{\partial L}\left(\frac{dL}{dt}\right) + \frac{\partial Y}{\partial E}\left(\frac{dE}{dt}\right)$$
$$\frac{dY/dt}{Y} = \frac{1}{Y}\frac{\partial Y}{\partial K}\left(\frac{dK}{dt}\right)\left(\frac{K}{K}\right) + \frac{1}{Y}\frac{\partial Y}{\partial L}\left(\frac{dL}{dt}\right)\left(\frac{L}{L}\right) + \frac{1}{Y}\frac{\partial Y}{\partial E}\left(\frac{dE}{dt}\right)\left(\frac{E}{E}\right)$$
$$= \frac{K}{Y}\frac{\partial Y}{\partial K}\frac{(dK/dt)}{K} + \frac{L}{Y}\frac{\partial Y}{\partial L}\frac{(dL/dt)}{L} + \frac{E}{Y}\frac{\partial Y}{\partial E}\frac{(dE/dt)}{E}$$
$$gr(Y) = \frac{K}{Y}\frac{\partial Y}{\partial K}gr(K) + \frac{L}{Y}\frac{\partial Y}{\partial L}gr(L) + \frac{E}{Y}\frac{\partial Y}{\partial E}gr(E)$$

In competitive market $\frac{\partial Y}{\partial K} = r$ and $\frac{\partial Y}{\partial L} = w$. K, Y, L and their growth rates can be measured directly.

$$gr(Y) = \frac{K}{Y}rgr(K) + \frac{L}{Y}wgr(L) + \frac{E}{Y}\frac{\partial Y}{\partial E}gr(E)$$
$$gr(Y) - \frac{K}{Y}rgr(K) - \frac{L}{Y}wgr(L) = \frac{E}{Y}\frac{\partial Y}{\partial E}gr(E) = Solow\ Residual$$

Growth Accounting Problems

- The Solow residual is defined as per-capita economic growth above the rate of per-capita capital stock growth, so its detection indicates that there must be some contribution to output other than advances in industrializing the economy
- The Solow residual measures total factor productivity, but is normally attached to the labour variable in the macroeconomy because return on investment doesn't change much in time or between developing nations
- Capital stock: measured with much error and leads to errors in the residual
- Labour quality: hard to properly account for the quality of labour, e.g. skills and education
- Example: Solow Computer Productivity Paradox, probably measurement error
- Alwyn Young's Study of East Asian Growth found that output growth was largely due to factor accumulation and not technology. However, a critique of Young was provided by Klenow, who argued that the focus should be on output per worker not output, and that as technology increases, capital also increases. Correcting for these effects, he found that in most of the Tigers, growth was driven by technology

Overlapping Generations Model

Purpose

Extends the Solow-Swan model by endogenising savings and permitting comparisons based on different preference utility models

Definition of Terms

$$N_t = \text{number of people born at time } t$$
$$c_y = \text{consumption of people while young at } t$$
$$c_o = \text{consumption of people while old at } t+1$$
$$w_t = \text{wages of young people working at } t$$
$$r_{t+1} = \text{interest rate on capital at } t+1$$
$$a_{t+1} = \text{assets saved by person born at time } t$$
$$\alpha = \text{parameter describing time preference}$$

Identity Equations

$$N_{t+1} = (1+n)N_t$$
$$U_t = \alpha \log(c_{y,t}) + (1-\alpha)\log(c_{o,t})$$
$$a_{t+1} = w_t - c_{y,t}$$
$$c_{o,t+1} = (1+r_{t+1})\, a_{t+1}$$

Deriving Budget Constraint

Need to get wage in terms of variables that can be selected (i.e. consumption)

$$c_{o,t+1} = (1+r_{t+1})(w_t - c_{y,t})$$
$$c_{o,t+1} = w_t + w_t r_{t+1} - c_{y,t} - r_{t+1}c_{y,t}$$
$$c_{o,t+1} + c_{y,t} + r_{t+1}c_{y,t} = w_t + w_t r_{t+1}$$
$$c_{o,t+1} + c_{y,t} + r_{t+1}c_{y,t} = w_t(1 + r_{t+1})$$
$$\frac{c_{o,t+1} + c_{y,t} + r_{t+1}c_{y,t}}{(1+r_{t+1})} = w_t$$
$$w_t = \frac{c_{o,t+1} + c_{y,t}(1+r_{t+1})}{(1+r_{t+1})}$$
$$w_t = \frac{c_{o,t+1}}{(1+r_{t+1})} + c_{y,t}$$

Utility Maximisation

$$\max U_t = \alpha \log(c_{y,t}) + (1-\alpha)\log(c_{o,t}) \ \text{ s.t. } \ w_t = \frac{c_{o,t+1}}{(1+r_{t+1})} + c_{y,t}$$
$$\max L = \alpha \log(c_{y,t}) + (1-\alpha)\log(c_{o,t}) + \lambda\left(w_t - \frac{c_{o,t+1}}{(1+r_{t+1})} - c_{y,t}\right)$$

Differentiate relative to choice variables $c_{y,t}$ and $c_{o,t}$:

$$\frac{\partial L}{\partial c_{y,t}} = \frac{\alpha}{c_{y,t}} - \lambda = 0$$
$$\frac{\partial L}{\partial c_{o,t}} = \frac{(1-\alpha)}{c_{o,t}} - \frac{\lambda}{(1+r_{t+1})} = 0$$
$$\frac{\partial L}{\partial \lambda} = w_t - \frac{c_{o,t+1}}{(1+r_{t+1})} - c_{y,t} = 0$$

$$0 = \frac{(1-\alpha)}{c_{o,t}} - \frac{\left(\frac{\alpha}{c_{y,t}}\right)}{(1+r_{t+1})}$$

$$\frac{(1-\alpha)}{c_{o,t}} = \frac{\left(\frac{\alpha}{c_{y,t}}\right)}{(1+r_{t+1})}$$

$$\frac{c_{o,t}}{(1-\alpha)} = \frac{(1+r_{t+1})}{\left(\frac{\alpha}{c_{y,t}}\right)}$$

$$c_{o,t} = \frac{(1+r_{t+1})(1-\alpha)}{\left(\frac{\alpha}{c_{y,t}}\right)}$$

$$c_{o,t} = (1+r_{t+1})(1-\alpha)\left(\frac{c_{y,t}}{\alpha}\right)$$

$$w_t = \frac{c_{o,t+1}}{(1+r_{t+1})} + c_{y,t}$$

$$w_t = \frac{(1+r_{t+1})(1-\alpha)\left(\frac{c_{y,t}}{\alpha}\right)}{(1+r_{t+1})} + c_{y,t}$$

$$= (1-\alpha)\left(\frac{c_{y,t}}{\alpha}\right) + c_{y,t}$$

$$= \left(\frac{c_{y,t}}{\alpha} - \alpha\frac{c_{y,t}}{\alpha}\right) + c_{y,t}$$

$$= \left(\frac{c_{y,t}}{\alpha} - c_{y,t}\right) + c_{y,t}$$

$$w_t = \frac{c_{y,t}}{\alpha}$$

$$c_{y,t} = w_t\alpha$$

$$c_{o,t} = (1+r_{t+1})(1-\alpha)\left(\frac{c_{y,t}}{\alpha}\right)$$

$$c_{o,t} = (1+r_{t+1})(1-\alpha)\left(\frac{w_t\alpha}{\alpha}\right)$$

$$c_{o,t} = (1+r_{t+1})(1-\alpha)w_t$$

Production Function

$$Y_t = AK_t^{\beta}N_t^{1-\beta}$$

$$r_t = \frac{\partial Y_t}{\partial K_t} = A\beta K_t^{\beta-1}N_t^{1-\beta}$$

$$w_t = \frac{\partial Y_t}{\partial L_t} = A(1-\beta)K_t^{\beta}N_t^{-\beta}$$

Knowing that $k_t = \frac{K_t}{N_t}$

$$r_t = \frac{\partial Y_t}{\partial K_t} = A\beta k_t^{\beta-1}$$

$$w_t = \frac{\partial Y_t}{\partial L_t} = A(1-\beta)k_t^{\beta}$$

Capital Stock Accumulation

Assume in the long run that there is full employment so $L_t = N_t$. Also assume that in every period, all old people consume all their saved capital and all their interest earnings. Thus:

$$K_{t+1} = N_t a_{t+1}$$

$$\begin{aligned} a_{t+1} &= (1-\alpha)w_t \\ w_t &= A(1-\beta)k_t^{\beta} \\ \therefore K_{t+1} &= N_t\left((1-\alpha)\left(A(1-\beta)k_t^{\beta}\right)\right) \\ &= A(1-\alpha)(1-\beta)k_t^{\beta}N_t \\ &= A(1-\alpha)(1-\beta)\left(\frac{K_t^{\beta}}{N_t^{\beta}}N_t\right) \\ K_{t+1} &= A(1-\alpha)(1-\beta)K_t^{\beta}N_t^{1-\beta} \end{aligned}$$

Population and Technological Growth

$$\begin{aligned} Y_t &= K_t^{\beta}(AN_t)^{1-\beta} \\ N_{t+1} &= (1+n)N_t \\ A_{t+1} &= (1+a)A_t \\ \frac{K_{t+1}}{A_{t+1}N_{t+1}} &= \frac{(1-\alpha)(1-\beta)K_t^{\beta}N_t^{1-\beta}A_t^{1-\beta}}{A_{t+1}N_{t+1}} \\ &= \frac{(1-\alpha)(1-\beta)K_t^{\beta}N_t^{1-\beta}A_t^{1-\beta}}{(1+a)A_t(1+n)N_t} \\ &= \frac{(1-\alpha)(1-\beta)K_t^{\beta}N_t^{-\beta}A_t^{-\beta}}{(1+a)(1+n)} \\ &= \frac{(1-\alpha)(1-\beta)K_t^{\beta}}{(1+a)(1+n)N_t^{\beta}A_t^{\beta}} \\ k_{t+1} &= \frac{(1-\alpha)(1-\beta)k_t^{\beta}}{(1+a)(1+n)} \end{aligned}$$

Steady state when $k_{t+1} = k_t = k^*$

$$k^* = \frac{K}{AN} = \left(\frac{(1-\alpha)(1-\beta)}{(1+a)(1+n)}\right)^{\frac{1}{1-\beta}}$$

Note that $\frac{1}{1-\beta} > 1$

Comparative Statics

- Long-run per-capita growth depends solely upon a, technological growth
- Lower α (more patient) increases steady-state
- Lower n (slower population growth) increases steady-state
- Lower a (slower population growth) increases steady-state

Fully Funded Social Security System

Young workers are taxed an amount equal toτ and receive a return of $(1 + r_{t+1})\tau$

$$s_{t+1} = w_t - c_t - \tau$$
$$c_t = w_t - \tau - s_{t+1}$$

$$\begin{aligned} c_{t+1} &= (1 + r_{t+1})\tau + (1 + r_{t+1})s_{t+1} \\ &= (1 + r_{t+1})(\tau + s_{t+1}) \\ &= (1 + r_{t+1})(\tau + w_t - c_t - \tau) \\ &= (1 + r_{t+1})(w_t - c_t) \end{aligned}$$

This is the same as the original consumption when old, thus lifetime budget constraint is unchanged. Hence, capital and output per worker are unchanged, assuming that individuals make rational saving decisions.

Pay as You Go Social Security System

Young workers are taxed an amount equal to τ and receive a return of $(1 + n)\tau$, which depends upon how much population has increased (n) since the last time period

$$\begin{aligned} s_{t+1} &= w_t - c_t - \tau \\ c_t &= w_t - \tau - s_{t+1} \\ c_{t+1} &= (1 + n)\tau + (1 + r_{t+1})s_{t+1} \\ &= (1 + n)\tau + (1 + r_{t+1})(w_t - c_t - \tau) \\ \frac{c_{t+1}}{1 + r_{t+1}} &= \frac{(1 + n)\tau}{1 + r_{t+1}} + (w_t - c_t - \tau) \end{aligned}$$

This last line represents the present value of consumption in time $t + 1$

$$\begin{aligned} pv(c_{t+1}) + c_t &= w_t - \tau - (w_t - c_t - \tau) + \frac{(1 + n)\tau}{1 + r_{t+1}} + (w_t - c_t - \tau) \\ &= w_t - \tau + \left(\frac{1 + n}{1 + r_{t+1}}\right)\tau \end{aligned}$$

If $n > r_{t+1}$ then $\left(\frac{1+n}{1+r_{t+1}}\right) > 1$, and hence the pay-as-you go system increases the present value of consumption. If $n < r_{t+1}$ then the pay-as-you go system reduces present value of consumption.

This is essentially occurring because the golden rate of savings depends only on the production function, whereas in these investment models we have been assuming that savings also depend upon preferences (esp. discount rate). This will produce dynamic inefficiency if we assume that the interest rate is the 'correct' discount rate.

Endogenous Growth

Basic Idea of the Romer Model

- Technological growth depends upon resources devoted to research and development, naturally subject to institutions and other factors
- Romer thus proposed a model of a two-sector economy
- Sector 1: Standard sector uses factors of production to produce output
- Sector 2: Technology producing sector uses factors of production to generate technology
- Labour is a scarce factor that must be allocated between these two sectors

Factor Accumulation

- Same as the Solow Model, but irrelevant to steady state levels of growth

$$
\begin{aligned}
Y_t &= K_t^{\alpha}\left(A_t L_{Y,t}\right)^{1-\alpha} \\
K_{t+1} &= (1-\delta)K_t + s_K Y \\
L_{t+1} &= (1+n)L_t
\end{aligned}
$$

Technology Accumulation

- Here ϕ represents the relationship between the level of technology and ease of producing new technology: greater than zero indicates increasing returns, and less decreasing returns

$$
\begin{aligned}
A_{t+1} - A_t &= cA_t^{\phi} L_{A,t} \\
L_t &= L_{A,t} + L_{Y,t} \\
L_t &= s_r L + L_{Y,t}
\end{aligned}
$$

$$
\begin{aligned}
g &= \frac{A_{t+1} - A_t}{A_t} = cA_t^{\phi-1} L_{A,t} \\
\textit{at steady state } g_t &= g_{t+1} \\
cA_t^{\phi-1} L_{A,t} &= cA_{t+1}^{\phi-1} L_{A,t+1} \\
A_t^{\phi-1} L_{A,t} &= [A_t(1+g)]^{\phi-1}\left[L_{A,t}(1+n)\right] \\
1 &= (1+g)^{\phi-1}(1+n) \\
\log(1) &= (\phi-1)\log(1+g) + \log(1+n) \\
0 &= (\phi-1)\log(1+g) + \log(1+n) \\
&\approx (\phi-1)g + n \\
-n &= (\phi-1)g \\
g &= \frac{-n}{\phi-1} \\
g &= \frac{n}{1-\phi}
\end{aligned}
$$

$$
\begin{aligned}
\textit{Alternatively}: 1 &= (1+g)^{\phi-1}(1+n) \\
\frac{1}{1+n} &= (1+g)^{\phi-1} \\
1+g &= \left(\frac{1}{1+n}\right)^{\frac{1}{\phi-1}} \\
g^* &= \left(\frac{1}{1+n}\right)^{\frac{1}{\phi-1}} - 1
\end{aligned}
$$

Implications and Empirics

- In the short run, growth increases with ϕ, L_A and n
- If $\phi > 1$, growth will approach infinity in the long-run
- If $\phi < 1$, long-run growth depends upon the value of ϕ and increases with n
- If $\phi = 0$, long-run growth is equal to population growth
- Kremer in his analysis plotted a positive correlation between levels of world population and rate of population growth (this being a proxy for technological growth), thereby apparently confirming the Romer model
- Of course, this is dependent upon the assumption that n is a valid proxy for g; otherwise all he has proven is that population growth is exponential

How Knowledge is Different

- Knowledge is central to technological advancement and is fundamentally different from other factors of production largely because it is both non-rivalrous and non-excludable
- Romer discusses three crucial aspects of growth:
- Technological growth is due to accumulation of ideas
- Accumulation of ideas is driven by incentives (e.g. lower interest rates increases returns)
- New ideas are costly to produce but have characteristics of public goods
- His conclusion was that models of growth required imperfect competition, as without imperfect competition, there would be little incentive to innovate

The Role of Imperfect Competition

- Will societies allocate resources to research efficiently?
- It is very hard to say, given the positive knowledge spillovers, negative congestion externalities in research (duplication of effort), and that research incentives are determined by monopoly profits but exclude benefits to the consumer
- Contrast between static and dynamic efficiency gains: in a static setting, monopolies are unambiguously welfare reducing, but in a dynamic setting, the ability to earn monopoly profits encourages research
- Historical examples of the negative effects of patent protection with James Watt and the development of the steam engine: progress in steam technology occurred more rapidly after patent expiry than before

Growth and Institutions

Growth Regressions

- Multicollinearity: most of the potential determinants of growth are correlated with each other and imperfectly measured, making it hard to figure out which is the true determinant, especially if some variables are omitted from the model
- Heterogeneity between countries: what makes you think there is a single relationship between, say, education and growth in all countries?
- Robustness: hard to find robust results that do not depend upon the particular construction of the model and variable selection
- Variable selection: not clear how to select which variables to include in the regression given the large number of possibilities
- Causality versus correlation: a parameter or variable is said to be endogenous when there is a correlation between the parameter or variable and the error term; makes it hard to separate cause from effect because of a feedback loop between the variables

Factor Prices and Interest Rates

- If physical technology is relatively similar across countries, then interest rates should be higher in poorer countries, and large investment flows should move from rich to poor countries

$$\frac{r_{India}}{r_{USA}} = \frac{A\beta k_{In}^{\beta-1}}{A\beta k_{US}^{\beta-1}}$$

$$= \frac{k_{In}^{\beta-1}}{k_{US}^{\beta-1}}$$

$$= \left(\frac{k_{In}^{\beta}}{k_{US}^{\beta}}\right)^{\frac{\beta-1}{\beta}}$$

$$= \left(\frac{y_{In}}{y_{US}}\right)^{\frac{\beta-1}{\beta}}$$

- Empirical evidence suggests that $\beta \approx 0.4$, and GDP per capita in the US is about ten times that of India, therefore:

$$\frac{r_{India}}{r_{USA}} = \left(\frac{1}{10}\right)^{-\frac{0.6}{0.4}}$$
$$= (10)^{\frac{3}{2}}$$
$$= \sqrt{10^3}$$
$$= \sqrt{1000}$$
$$\approx 32$$

- This indicates that interest rates in India should be able thirty times those in the US if productivity is the same
- The fact that this is not the case implies constant productivity assumption is incorrect

Hall and Jones Growth Accounting

- Growth accounting traditionally focused on growth rates, but similar analysis may be done that focuses on levels of output
- Note that here H (human capital) = L (number of workers) * h (education levels)

$$Y = K^{\alpha}(AH)^{1-\alpha}$$
$$Y^{\alpha}Y^{1-\alpha} = K^{\alpha}(AH)^{1-\alpha}$$
$$Y^{1-\alpha} = \frac{K^{\alpha}}{Y^{\alpha}}(AH)^{1-\alpha}$$
$$Y = \left(\frac{K^{\alpha}}{Y^{\alpha}}(AH)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}$$
$$Y = \left(\left(\frac{K}{Y}\right)^{\alpha}(AhL)^{1-\alpha}\right)^{\frac{1}{1-\alpha}}$$
$$= \left(\frac{K}{Y}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} AhL$$
$$y = \left(\frac{K}{Y}\right)^{\left(\frac{\alpha}{1-\alpha}\right)} Ah$$

- In steady state the capital-output ratio $\frac{K}{Y}$ is proportional to the saving-investment rate
- Therefore, this decomposition reveals output per worker as a function of capital intensity, human capital per worker and productivity

- Using this model, Hall and Jones found that despite large differences in output per worker across countries, there are relatively small differences in capital and education
- This implies that differences in output must be due to differences in productivity

Social Infrastructure

- Obviously relevant for growth: Social infrastructure determines incentives; Incentives determine economic choices; Economic choices determine inputs and productivity; Inputs and productivity determine output per worker
- Conducive social infrastructure provides private incentives that match closely with social incentives. This generates economic choices close to those that would maximise social welfare
- Very complex and difficult to model formally
- Research focuses on more detailed case studies
- Example: compare the shock therapy in Eastern Europe to gradual reforms in china

Production or Predation

- Murphy, Shliefer, and Vishny (1991) argued that lawyers mostly redistribute profit between people, while engineers are actually value creating
- They found that faster growing countries tended to have more engineers per capita compared to lawyers than slower growing states, thought of course one could posit many possible reasons why this may be so
- Though their specific analysis is questionable, the basic point is that societies will be better off if more resources are devoted to generating wealth than protecting it from others

The Origins of Social Infrastructure

- Acemoglu, Johnson and Robinson (2004) argue that political power and preferences determine institutions
- In particular, institutions determine the distribution of wealth, and the distribution of wealth determines political power
- Individuals are often unable to agree on efficient institutions (in contrast to what happens in markets) is because often, people with political power find it is not in their best interest to provide a more equitable distribution of outcomes, or more efficient institutiosn
- This may weaken their political power and leave them worse off in the future, even if society as a whole is much better off
- An Example: Monarchies During the Middle Ages tended to protect their own property rights, but had little incentive to protect property rights of others, thereby resulting in slower growth

Evidence that Institutions Matter

- Interest rates: If productivity was the same across countries, India should have much higher interest rates than the US: over 30 times higher (it doesn't)
- Hall and Jones (1999): Found large differences in output per worker across countries, but relatively small differences in capital and education. This implies differences in output must be due to differences in productivity
- Large migrations: Irish famine, early 20th century migration to USA, and post-war migration from East Germany all associated with rise in incomes of receiving countries

- Individual migration: immigrants to the US from poor countries earn about half of native-born Americans of the same age, sex and years of schooling
- Geographic comparisons: North and South Korea, US-Mexico border, Taiwan and China, East and West Germany
- Napoleonic Code: conquest of Europe introduced French institutions into much of Europe, such that conquered countries urbanised and industrialised sooner than other regions
- Direct measurement: various direct measures of institutional quality, such as property right protection, quality of governance, language spoken, etc, generally yield strong results

Business Cycles

Expectations and the Cobb-Webb Model

Deriving Equilibrium Price

Price depends upon expectations since supply within the market determined by expectations. Note that in equilibrium price and price expectations need not be equal unless we assume myopic expectations. However, in equilibrium it will be the case that demand equals supply and prices are stable. Of course, the model need not converge to an equilibrium.

$$\begin{aligned} d_t &= demand \\ s_t &= supply \\ v_1, v_2 &= random\ elements \\ E(v_1) &= E(v_2) = 0 \end{aligned}$$

$$\begin{aligned} d_t &= m_I - m_p p_t + v_{1,t} \\ s_t &= r_I - r_p p_t^e + v_{2,t} \end{aligned}$$

$$\begin{aligned} d_t &= s_t \\ m_I - m_p p_t + v_{1,t} &= r_I - r_p p_t^e + v_{2,t} \\ m_p p_t &= r_I - r_p p_t^e + v_{2,t} - m_I - v_{1,t} \\ p_t &= \frac{r_I - m_I}{m_p} - \frac{r_p}{m_p} p_t^e + \frac{v_{2,t} - v_{1,t}}{m_p} \\ &= \mu + \alpha p_t^e + n_t \end{aligned}$$

Myopic Expectations

Problems: Individuals make systematic mistakes, possibility for arbitrage as price converges

$$\begin{aligned} & p_t^e = p_{t-1} \\ & \therefore p_t = \mu + \alpha p_{t-1} + n_t \\ & p_{t+1} = \mu + \alpha(\mu + \alpha p_{t-1} + n_t) + n_{t+1} \end{aligned}$$

Converges if $|\alpha| < 1$

Rational Expectations

With myopic expectations, price deviations from expected are forecastable. With rational expectations, price deviations from expected are non-forecastable. This removes the ability of individuals to arbitrage or for systematic mistakes.

$$p_t^e = E_{t-1}p_t$$

$$p_t = \mu + \alpha p_t^e + n_t$$
$$p_t = \mu + \alpha E_{t-1}p_t + n_t$$
$$E_{t-1}(p_t) = E_{t-1}(\mu + \alpha E_{t-1}p_t + n_t)$$
$$E_{t-1}(p_t) = E_{t-1}(\mu) + \alpha E_{t-1}(E_{t-1}p_t) + E_{t-1}(n_t)$$
$$E_{t-1}p_t = \mu + \alpha E_{t-1}p_t$$
$$E_{t-1}p_t - \alpha E_{t-1}p_t = \mu$$
$$E_{t-1}p_t(1-\alpha) = \mu$$
$$E_{t-1}p_t = \frac{\mu}{1-\alpha} = p_t^e$$
$$\therefore p_t = \mu + \alpha\left(\frac{\mu}{1-\alpha}\right) + n_t$$

Lucas Supply model

Based on the idea that nominal wage contracts are set in advance and depend upon expectations of future prices. Contracts are set so if price equals expected, output equals natural rate. If price is higher than expected, implies real wages are lower, so there is an increase in y. If price is lower than expected, implies real wages are higher, therefore a decrease in y.

$$y_n = natural\ level\ of\ output$$
$$u_t = productivity\ shock$$
$$m_t = indication\ of\ monetary\ policy\ or\ money\ supply$$
$$m_t = \mu_0 + e_t$$
$$e_t = stochastic\ change\ in\ MP$$

$$\text{Aggregate Demand: } y_t = \beta_0 + \beta_1(m_t - p_t)$$
$$\text{Aggregate Supply: } y_t = y_n + \alpha_1(p_t - E_{t-1}p_t) + u_t$$

$$\text{aggregate supply} = \text{aggregate demand}$$
$$\beta_0 + \beta_1(m_t - p_t) = y_n + \alpha_1(p_t - E_{t-1}p_t) + u_t$$
$$\beta_0 + (\beta_1 m_t - \beta_1 p_t) = y_n + (\alpha_1 p_t - \alpha_1 E_{t-1}p_t) + u_t$$
$$\beta_0 - y_n + \beta_1 m_t + \alpha_1 E_{t-1}p_t - u_t = \alpha_1 p_t + \beta_1 p_t$$
$$\beta_0 - y_n + \beta_1 m_t + \alpha_1 E_{t-1}p_t - u_t = p_t(\alpha_1 + \beta_1)$$
$$p_t = \frac{\beta_0 - y_n + \beta_1 m_t + \alpha_1 E_{t-1}p_t - u_t}{\alpha_1 + \beta_1}$$

$$E_{t-1}p_t = \frac{E_{t-1}\beta_0 - E_{t-1}y_n + E_{t-1}\beta_1 m_t + E_{t-1}\alpha_1 E_{t-1}p_t - E_{t-1}u_t}{E_{t-1}\alpha_1 + E_{t-1}\beta_1}$$
$$E_{t-1}p_t = \frac{\beta_0 - y_n + \beta_1 E_{t-1}m_t + \alpha_1 E_{t-1}p_t}{\alpha_1 + \beta_1}$$

Policy Irrelevance Hypothesis

First, work out rational expectations of price by subtracting expected price from price

$$p_t - E_{t-1}p_t = \frac{\beta_0 - y_n + \beta_1 m_t + \alpha_1 E_{t-1}p_t - u_t}{\alpha_1 + \beta_1} - \frac{\beta_0 - y_n + \beta_1 E_{t-1}m_t + \alpha_1 E_{t-1}p_t}{\alpha_1 + \beta_1}$$

$$= \frac{\beta_1 m_t + \beta_1 E_{t-1}m_t - u_t}{\alpha_1 + \beta_1}$$

$$= \frac{\beta_1 (m_t - E_{t-1}m_t) - u_t}{\alpha_1 + \beta_1}$$

$$p_t - E_{t-1}p_t = \frac{\beta_1}{\alpha_1 + \beta_1}(m_t - E_{t-1}m_t) - \frac{u_t}{\alpha_1 + \beta_1}$$

Substitute this (rational expectations of future price) into supply curve

$$y_t = y_n + \alpha_1(p_t - E_{t-1}p_t) + u_t$$

$$y_t = y_n + \alpha_1\left(\frac{\beta_1}{\alpha_1 + \beta_1}(m_t - E_{t-1}m_t) - \frac{u_t}{\alpha_1 + \beta_1}\right) + u_t$$

$$= y_n + \frac{\alpha_1\beta_1}{\alpha_1 + \beta_1}(m_t - E_{t-1}m_t) - \frac{\alpha_1 u_t}{\alpha_1 + \beta_1} + \frac{u_t\alpha_1 + u_t\beta_1}{\alpha_1 + \beta_1}$$

$$= y_n + \frac{\alpha_1\beta_1}{\alpha_1 + \beta_1}(m_t - E_{t-1}m_t) + \frac{\beta_1}{\alpha_1 + \beta_1}u_t$$

Thus, output depends upon the natural rate of output, supply shocks, and deviations in money supply from expected. Crucially, it does not depend upon expected changes in money supply, as firms and unions already take this into account when making supply decisions. This occurs because given rational expectations, the expected value of prices appears in the supply curve (because it affects real labour costs), and expected monetary policy interventions in turn affect expected value of prices.

However, rational expectations is an extreme assumption that is unlikely to be literally true, as rigidities in the economy may still generate policy effects. For example, if some contracts are fixed for some period of time and are negotiated in nominal terms.

The Lucas Critique

- It is naïve to try to predict the effects of a change in economic policy entirely on the basis of relationships observed in historical data, especially highly aggregated historical data
- Parameters of large macroeconomic models were not structural, i.e. not policy-invariant, they would necessarily change whenever policy (the rules of the game) was changed
- One important application of the critique is its implication that the historical negative correlation between inflation and unemployment, known as the Phillips Curve, could break down if the monetary authorities attempted to exploit it. Permanently raising inflation in hopes that this would permanently lower unemployment would eventually cause firms' inflation forecasts to rise, altering their employment decisions.

Life Cycle Model of Consumption

Keynesian Consumption Function

- Standard approach in introductory and intermediate macroeconomics is to assume that consumption is a simple function of income: $C = a + bY$

- However, this does not explain the empirical observation that income is much more volatile than consumption, nor does it explain individual behaviour very well
- An alternative model, the Lifetime Income Hypothesis, is a variant of the OLG model

Intertemporal Choice Consumption Model

Budget Constraints

$$y_1 = c_1 + a_2$$
$$c_1 = y_1 - a_2$$

$$y_2 = c_2 - (1+r)a_2$$
$$c_2 = y_2 + (1+r)a_2$$

$$c_2 = y_2 + (1+r)a_2$$
$$c_2 = y_2 + (1+r)(y_1 - c_1)$$
$$c_2 = y_2 + (1+r)y_1 - (1+r)c_1$$
$$c_2 + (1+r)c_1 = y_2 + (1+r)y_1$$
$$\frac{c_2}{(1+r)} + c_1 = \frac{y_2}{(1+r)} + y_1$$
$$\text{present value of c} = \text{present value of y}$$

Consumer Preferences

Assume Non-satiation, Diminishing returns to consuming in a single period and that Current and future consumption are normal goods

$$U = \log(c_1) + \frac{1}{1+\rho}\log(c_2)$$
$$\text{where } \rho \text{ } is \text{ a time preference parameter}$$

Calculus of Intertemporal Choice

Can solve with Langrangian or substitution to get the Euler equation:

$$\max\left\{U(c_1) + \frac{1}{1+\rho}U(c_2)\right\} \; s.t. \; \frac{c_2}{(1+r)} + c_1 = \frac{y_2}{(1+r)} + y_1$$
$$\max\left\{U(c_1) + \tfrac{1}{1+\rho}U(c_2)\right\} = \max\left\{\mathrm{U}(c_1) + \tfrac{1}{1+\rho}\mathrm{U}(y_2 + (1+r)y_1 - (1+r)c_1)\right\}$$
$$\frac{\partial U}{\partial c_1} = U'(c_1) - \frac{1+r}{1+\rho}U'(c_2) = 0$$
$$\therefore U'(c_1) = \frac{1+r}{1+\rho}U'(c_2)$$
$$c_1 = \frac{1+r}{1+\rho}c_2$$
$$\text{Marginal benefit of consumption today} = \text{Marginal benefit of consumption tomorrow}$$

Incorporating Uncertainty

Assume that future income y_2 is a random variable to get the stochastic Euler equation

$$\max\left\{U(c_1) + \frac{1}{1+\rho}U(c_2)\right\} \; s.t. \; \frac{c_2}{(1+r)} + c_1 = \frac{y_2}{(1+r)} + y_1$$

$$\max\left\{E[U(c_1)] + \frac{1}{1+\rho}E[U(y_2 + (1+r)y_1 - (1+r)c_1)]\right\}$$

$$\frac{\partial U}{\partial c_1} = U'(c_1) - E\left[\frac{1+r}{1+\rho}U'(c_2)\right] = 0$$

$$\therefore U'(c_1) = \frac{1+r}{1+\rho}E[U'(c_2)]$$

Marginal benefit of consumption today = Expected marginal benefit of consumption tomorrow

Permanent Income Hypothesis

Suppose that $\rho = r$

$$c_1 = \frac{1+r}{1+r}c_2$$

$$c_1 = c_2 = c$$

$$\therefore \frac{c}{(1+r)} + c = \frac{y_2}{(1+r)} + y_1$$

$$c\left(\frac{1}{1+r} + 1\right) = \frac{y_2}{(1+r)} + y_1$$

$$c = \frac{\frac{y_2}{(1+r)} + y_1}{\left(\frac{2+r}{1+r}\right)}$$

$$c = \left(\frac{1+r}{2+r}\right)\left(\frac{y_2}{1+r} + y_1\right) = y_p$$

Implications of Permanent Income

- Present consumption is the same as future consumption
- Consumption depends upon both current and future income
- Intuition: When extra income arrives, consume some today but save some for tomorrow. If we expect extra income arrives tomorrow, consume some today
- Has better empirical success relative to the simple Keynesian consumption function
- However, requires perfectly functioning credit markets (e.g. no information asymmetry)
- This implies that temporary tax cuts will have little or no effect on current consumption, as they do not change total lifetime income, while permanent tax cuts will have an effect

Ricardian Equivalence

- Assumes perfectly functioning credit markets
- Ignores the fact that government taxation may have distortionary effects on output
- People do not have complete knowledge of government spending and taxation decisions

Consumption as a Random Walk

- The stochastic Euler equation states that $c_1 = E(c_2)$
- If consumers form their estimates of permanent income rationally, then this error (the error in estimating future income) must be serially uncorrelated
- If the random walk hypothesis is correct, no variable other than current consumption should be useful predict future consumption, as it already contains all necessary information
- Robert Hall carried out a test of this hypothesis assuming quadratic preferences, and found (in support of the theory) that income does not predict future consumption when controlling for present consumption

- However, in contradiction with the theory, wealth (as proxied by stock prices) did predict consumption
- Since Hall's test was a joint test of all assumptions of his model, including rational expectations, quadratic preferences and random walk theory, we cannot exclusively rule out the strong random walk hypothesis

Neoclassical Theory of Investment

Basic Definitions

Investment comprises about twenty percent of GDP, but it is generally more volatile than either output or consumption

$$\begin{aligned}
&I_t = K_{t+1} - (1-\delta)K_t \\
&P_I: price\ of\ capital \\
&P: price\ of\ output \\
&K_0: initial\ capital\ stock \\
&K = K_0 + I \\
&\delta: depreciation\ rate \\
&Y = f_K K: output\ (linear\ function\ of\ K)
\end{aligned}$$

Costs of Investment

The MC of investment is assumed to increase as I increases, owing to increasing disruption of production:

$$C(I) = P_I I + \frac{a}{2} P_I I^2$$

Profit Maximising Firm

$$\text{Note that: } \sum_{x=0}^{\infty} x^n = \frac{1}{1-x}$$

$$\begin{aligned}
\text{Profit} &= \frac{PY}{(1+r)} + \frac{PY(1-\delta)}{(1+r)^2} + \cdots + \frac{PY(1-\delta)^t}{(1+r)^{t+1}} - \left(P_I I + \frac{a}{2} P_I I^2\right) \\
&= \frac{PY}{(1+r)}\left(1 + \frac{(1-\delta)}{(1+r)} + \cdots + \left(\frac{1-\delta}{1+r}\right)^t\right) - P_I I - \frac{a}{2} P_I I^2 \\
&= \frac{PY}{1+r}(1 + x + \cdots + x^t) - P_I I - \frac{a}{2} P_I I^2 \\
&= \frac{PY}{1+r}\left(\frac{1}{1-x}\right) - P_I I - \frac{a}{2} P_I I^2 \\
&= \frac{PY}{1+r}\left(\frac{1}{1 - \frac{(1-\delta)}{(1+r)}}\right) - P_I I - \frac{a}{2} P_I I^2 \\
&= \frac{PY}{(1+r) - (1-\delta)} - P_I I - \frac{a}{2} P_I I^2 \\
&= \frac{P f_K (K_0 + I)}{r + \delta} - P_I I - \frac{a}{2} P_I I^2
\end{aligned}$$

$$\text{max(profit)}: \frac{d(\pi)}{dI} = \frac{P f_K}{r+\delta} - P_I - a P_I I$$

$$0 = \frac{Pf_K}{r+\delta} - P_I - aP_I I$$

$$aP_I I = \frac{Pf_K}{r+\delta} - \frac{P_I(r+\delta)}{r+\delta}$$

$$I = \frac{\frac{Pf_K - P_I(r+\delta)}{r+\delta}}{aP_I}$$

$$= \frac{Pf_K - P_I(r+\delta)}{aP_I(r+\delta)}$$

$$= \frac{1}{a}\left(\frac{Pf_K}{P_I(r+\delta)} - 1\right)$$

$$q = \frac{Pf_K}{P_I(r+\delta)} = \frac{MB_I}{MC_{I,I=0}}$$

Implications of the Theory

- Greater adjustment costs (a) reduces investment
- An increase in productivity (Y) increases investment
- An increase in the relative price of output to capital goods $(P > P_I)$ increases investment
- Increases in the real interest rate (r) and the depreciation rate (δ) lower investment

Limitations to Theory

- Assumes (fairly unrealistically) constant marginal product of capital
- Does not incorporate taxation of profits
- Does not incorporate expectations of future profits
- In reality, investment decisions evolve over time

Empirical Tests of the Theory

- Testing the theory requires knowledge of the marginal value of investment relative to the marginal cost of investment (q)
- Unfortunately, this is generally unobservable in data that is readily available
- Instead, we can use as a proxy the average value of investment divided by the average cost of the investments (average Q)

$$Q = \frac{\text{total market share value}}{\text{total cost of capital}}$$

- The theory implies that investment should only be determined by factors that determine q
- This means that if we regress investment on q and a bunch of other parameters (for data from many firms), only the coefficient on q should be significant
- Results of these tests show that the coefficient on Q is important, however, other factors are also important in determining investment, including cash flow and level of output
- This may indicate limitations to the model (e.g. importance of credit constraints and large fixed costs of investment), or limitations of using Q as a proxy for q

Real Business Cycle Theory

Basic Idea

- Fluctuations in productivity drive economic fluctuations and wage-price rigidities are not important; no role for money

- RBC theory sees recessions and periods of economic growth as the efficient response to exogenous changes in the real economic environment, therefore no need for government intervention
- Economy is always assumed to be in equilibrium

Model Assumptions

- Assumes two-period decision model
- Individuals in the economy have two decisions
  - 1. A labour supply decision
  - 2. A consumption and saving decision
- Firms in the economy have two decisions
  - 1. A labour demand decision
  - 2. An investment decision
- Model aggregate consumption in a manner similar to individual consumption decision and aggregate investment in a manner similar to individual investment decision (representative agent assumption)
- Firm and consumer heterogeneity is much more difficult to model, and requires more advanced concepts

Leisure-Labour Supply Decision

$$U(C) = utility\ from\ consumption$$
$$C = wL$$
$$v(L) = disutilty\ from\ labour$$

$$\max\big(U(C) - v(L)\big) = \frac{d\big(U(wL) - v(L)\big)}{dL}$$
$$U'^{(wL)}w - v'^{(L)} = 0$$
$$w = \frac{v'(L)}{U'(C)}$$

The marginal rate of substitution between consumption and leisure is equal to the wage rate

Consumption-Saving Decision

Trade-off between consuming today and consuming in the future. In equilibrium (see above):

$$U'(c_1) = \frac{1+r}{1+\rho}[U'(c_2)]$$

Labour Demand Decision

The profit maximising firm will hire labour until:

$$\frac{\partial Y}{\partial L} = w$$

Investment Demand

Continue investment until the marginal product of capital is equal to the interest rate:

$$MP_k = r$$

Finding the Equilibrium

- Equilibrium conditions:
    - Labour demand is equal to labour supply $L = N$
    - Output is equal to consumption, investment and gov. spending $Y = C + I + G$
- Unlike Keynesian economics, there is no role for money or bond markets
- The wage rate adjusts in the labour market to ensure that demand is equal to supply
- The interest rate adjusts to ensure that output equals aggregate demand

Productivity Shocks

- Supply shocks can come from many sources, including improved technologies, new organisational techniques, changes in the prices of key factors of production (e.g. oil), natural disasters and weather, wars, and competitive market changes
- A positive productivity shock would increase the marginal product of labour and shifts out labour demand curve
- This increases the wage rate to induce greater labour supply, and hence increases the equilibrium level of employment
- Increased wages and labour supply increase wealth, thereby increasing consumption and saving across all periods; greater saving causes interest rates to fall and investment to increase
- This is consistent with the fact that consumption, income, investment and employment all increase during booms

Calibrating the Model

- Real Business Cycle models are difficult to estimate directly because there are so many parameters to be estimated with little data, and it is difficult to properly identify all the models and equations (underidentification)
- These problems often lead to a process of calibration whereby “reasonable" parameters are selected using prior or external information, such as datasets on micro response of labour supply to wages, the assumption that $\rho = r$, and the measured capital share of income related to the production function
- The model is then simulated using realistic shocks, derived from Solow residuals used for technology/productivity shocks
- The outcome of the model is then compared to actual data on bases such as standard deviation of output and correlation of output with other endogenous variables
- If the moments are matched, success. If not, calibration and model structure are adjusted until the data of interest is matched

Problems with Calibration

- It is also somewhat arbitrary in the exact construction of the model, selection of parameters and comparison to actual data
- Why would you consider matching the moments in the data a desirable property in a model that you know leaves out important features?
- Statistical tests are meaningless except in the context of an alternative. It may be the case that there are hundreds of different models that do equally well
- RBC theory seems to indicate that many deep recessions are periods of technical regress (especially the Great Depression or the recent troubles in Japan)

- If the Solow residual measures what RBC model says it does, then it should be uncorrelated with the political party of the President, millitary purchases, and oil price movements (once energy usage has been accounted for). But it is correlated with all those things (Hall)

Evaluation of RBC Theory

- Strengths: explains increases in employment, output, consumption and investment during booms, and to some extent the converse during busts
- Weakness: Wages are weakly procyclical in the data but strongly procyclical in the model, monetary factors do have some impact upon the real economy, and it is difficult to explain why productivity shocks occur or how financial crises cause them
- Even strongest supporters of RBC do not believe productivity fluctuations are the complete story – money still matters
- In particular, the 'voluntary unemployment' explanation of the RBC theory is inadequate to many

Financial Markets

Basic Financial Mechanisms

- Financial factors affect the ability of firms to raise funds for investment (direct effect)
- Financial downturn reduces confidence and expectations of future (indirect effect)
- Poor financial health damages real economy which worsens the state of the financial sector (feedback effect)

Crisis Phase 1: Creation of an Asset Bubble

- Additional credit becomes available thanks to easing of credit constraints (easy monetary policy), financial system deregulation (e.g. Scandinavia 1980s), or new product market innovation (introduction of tulips, mortgage backed securities)
- Initial price increases due to the increased purchasing power or new production possibilities
- In particular, greater uncertainty in the degree of credit expansion and price rises leads to an increase in the possible gains, thereby leading to an especially large increase in the prices of these risky assets, as under asymmetric information and moral hazard (e.g. of money managers) risky assets are preferred
- Later, asset price increases driven by continued purchases under the belief that prices will continue to rise
- More credit and more uncertainty both translate to larger bubble

Crisis Phase 2: Collapse of the Asset Price Bubble

- Eventually, asset prices stop rising (caused by financial or real shock)
- Borrowers whose valuation of these assets depended on them continuing to rise are unable to repay loans
- Banks accumulate bad loans affecting their ability to provide intermediation
- Complications in the case of currency crisis: Loss of faith in an economy leads to outflow of foreign investment, and hence fall in the currency
- Increased interest rates can be used to defend the currency but damaging to the domestic economy

Crisis Phase 3: Spread to the Real Economy

- Failure of intermediation affects other sectors of the economy, as firms find it difficult to access credit from illiquid or failing banks and financial institutions
- Households poor balance sheets also cause them to reduce consumption (wealth effect)
- Reduction in demand caused by failing investment and consumption reduces profits, damaging ability to repay loans and thus worsening the financial situation
- Key difference between housing (2008) and stock bubble (2001) is degree of leverage – more leverage makes the system less stable
- Debt-deflation: When economy starts contracting, prices fall, while debt contracts are specified in nominal contracts, thus the real value of debt rises, which worsens the situation

Financial Accelerator

- Adverse conditions in the real economy and financial markets mutually reinforce each other, leading to a feedback loop that propagates the financial and macroeconomic downturn
- For example, a shock to the financial sector reduces the value of collateral
- Firms require collateral to borrow, and banks need a certain reserve to grant loans
- Reduced value of collateral thus reduces investment through demand and supply channels
- Reductions in investment damage profits, which in turn further reduce asset values
- Meanwhile, as firms cut back production due to reduced investment demand, workers are laid off, thereby raising unemployment, reducing consumption demand, and hence further lowering asset price values

Agency Problems

- Borrower wants to invest but does so through an intermediary, e.g. banks, superannuation funds, etc
- Lenders cannot observe the riskiness of an investment
- This creates a moral hazard problem in that the actions of the intermediary may not be in the best interests of the lender
- In particular, intermediaries receive benefits of profits when risky investments go well
- However, intermediaries bear limited costs when risky investments perform poorly
- Thus, asymmetric payoffs imply that financial intermediaries will overvalue risky assets
- The riskier the asset, the greater the problem will be, so financial reforms and monetary policy should aim to keep credit growth rates predictable

The Role of Government

- Bubbles may be difficult to detect due to uncertain fundamentals; e.g. in October 1929 Irving Fisher said “Stock prices have reached what looks like a permanently high plateau”
- The Fed also believed increased financial innovation justified higher house prices (house prices have not fallen in Australia like they have in the US)
- Monetary policy deemed as less effective due to breakdown in financial intermediation and the zero lower bound on interest rates
- Also, monetary policy is a blunt instrument for dealing with bubbles
- Debate about effectiveness of government spending: can the government spend in a timely fashion? What about crowding out of investment? The fiscal debt burden?

The Diamond-Dybvig Model

This model provides a mathematical statement of the idea that an institution with long-maturity assets and short-maturity liabilities may be unstable.

Investment made at $t = 0$ costs one unit of money. Let r be the return on the asset at period t. Then: $r_1 = 0$ (as investment is cancelled and principle returned), and $r_2 = r > 1$.

Consumers face utility function dependent upon consumption in the second two periods: c_1, c_2. Important properties are that $U'(c) > 0$ and $U''(c) < 0$. All consumers at $t = 0$ invest and want to hold until maturity at $t = 2$.

However, some proportion θ of consumers experience an unlucky shock at $t = 1$ which forces them to cancel their investment. Unlucky individuals have $U = u(1)$ (as for them $c_2 = 0$), while lucky individuals have $U = \beta u(1 + r)$ (as for them $c_1 = 0$), where $\beta < 1$ is a discount factor $\left(\beta = \frac{1}{1+\rho}\right)$.

Budget constraint will represent the relationship between c_1 and c_2. Essentially, $\sum c_2$ of all lucky consumers will be equal to the return on the investment of all agents, minus the total return of unlucky agents who consume at $t = 1$. This is given by:

$$N(1-\theta)c_2 = N(1+r) - N\theta c_1(1+r)$$
$$(1-\theta)c_2 = (1+r)(1-\theta c_1)$$

$$\max[E(U)] = \max[\theta u(c_1) + (1-\theta)\beta u(c_2)] \text{ s.t.} (1-\theta)c_2 = r(1-\theta c_1)$$
$$c_2 = \left(\frac{(1+r)(1-\theta c_1)}{(1-\theta)}\right)$$
$$= \left(\frac{(1+r)}{(1-\theta)}(1-\theta c_1)\right)$$

$$\max[E(U)] = \max\left[\theta u(c_1) + (1-\theta)\beta u\left(\frac{(1+r)}{(1-\theta)}(1-\theta c_1)\right)\right]$$
$$\frac{\partial E(U)}{\partial c_1} = \theta u'(c_1) + (1-\theta)\frac{(1+r)}{(1-\theta)}(-\theta)\beta u\left(\frac{r(1-\theta c_1)}{(1-\theta)}\right)$$
$$0 = \theta u'(c_1) - (1+r)\theta\beta u'(c_2)$$
$$u'(c_1) = (1+r)\beta u'(c_2)$$

This implies that with optimal financial intermediation, $1 < c_1 < c_2 < R$, as opposed to the unmediated case where $c_1 = 1$ and $c_2 = R$.

Bank Runs and Instabilities

- Banks can achieve this optimum result by acting as financial intermediaries
- Pay a low interest rate on short-term deposits even though a short-term investment will not produce this rate of return
- Pay a higher interest rate for long-term deposits than short-term deposits but below the rate of return of a project that is completed
- Specifically, banks accept deposits and invest deposits in projects. Some investors will withdraw early (but the bank doesn't know who ex ante), and they are paid $c_1 > 1$

- Other investors will keep their money in for the full term, and they are paid $c_2 < R$ to cover early withdrawers and provide profits for the bank
- This will increase c_1 and reduce c_2 but leave everyone better off in expectation
- However, as early withdrawal is effectively creating a loss for the bank, if many individuals withdraw early, this will lead to large losses and banks will be unable to pay any deposits in the future
- Thus, If all other individuals withdraw early it is best for me to withdraw early, as if I wait, no assets will remain. Thus my utility depends upon the action of others; complementarity in decisions can lead to multiple equilibria (two in this case)
- Solutions to this Instability include deposit insurance, where guaranteed deposits imply no reason for run to occur
- Problems with deposit insurance include moral hazard (bank may undertake more risky behaviour) and adverse selection (could encourage investment in more risky banks)

International Macroeconomics

Current Account Intertemporal Approach

- Definition of the current account: $CA = X - M = S - I$
- Just as persons might want to borrow more in their early life, for example to acquire education which will enhance that persons income later on in life, so might countries want to borrow more internationally in a phase of economic development, in order to built up production capacity which may enhance growth and hence income later on
- This idea can be represented diagrammatically by an outward shift of the budget line representing the trade-off between current and future consumption (determined by the interest rate)
- The basic idea behind this is that utility might be increased by either borrowing money to increase consumption now at the expense of consumption tomorrow, or lending money now to increase consumption tomorrow at the expense of some consumption today
- Opening an economy to international financial flows permits these two possibilities, and so would likely make a country better off; at worst there will be no difference
- Examples: Natural disasters lead to larger current account deficits (reduction in current but not future output, so leads to borrowing); Japan, as a neutral country, had a large current account surplus during WW1 (owing to increase in exports during the war, hence more income now but no more later, so lend now to increase consumption later)

Investment and the Current Account

- More advanced models of the current account endogenous investment and saving decisions to produce endogenous models of output in multiple periods
- Investment determined by: Marginal product of capital and Productivity growth
- Savings determined by: Present income, expected future income, preferences such as the degree of patience, and demographics

Global Savings Glut

- Common idea: larger current account deficits are a cause for concern, and indeed current account surpluses in China and East Asia led to easy credit and housing bubble in the USA

- Exchange rate restrictions in China make exports more competitive, thereby increasing production and employment in China and reduces it in the USA
- Alternative view - China is providing a subsidy to US consumers
- If higher deficit implies greater productive investment then no need for undue concern
- Alternatively, if a higher deficit implies greater consumption or financing of asset price bubbles, then it could be problematic

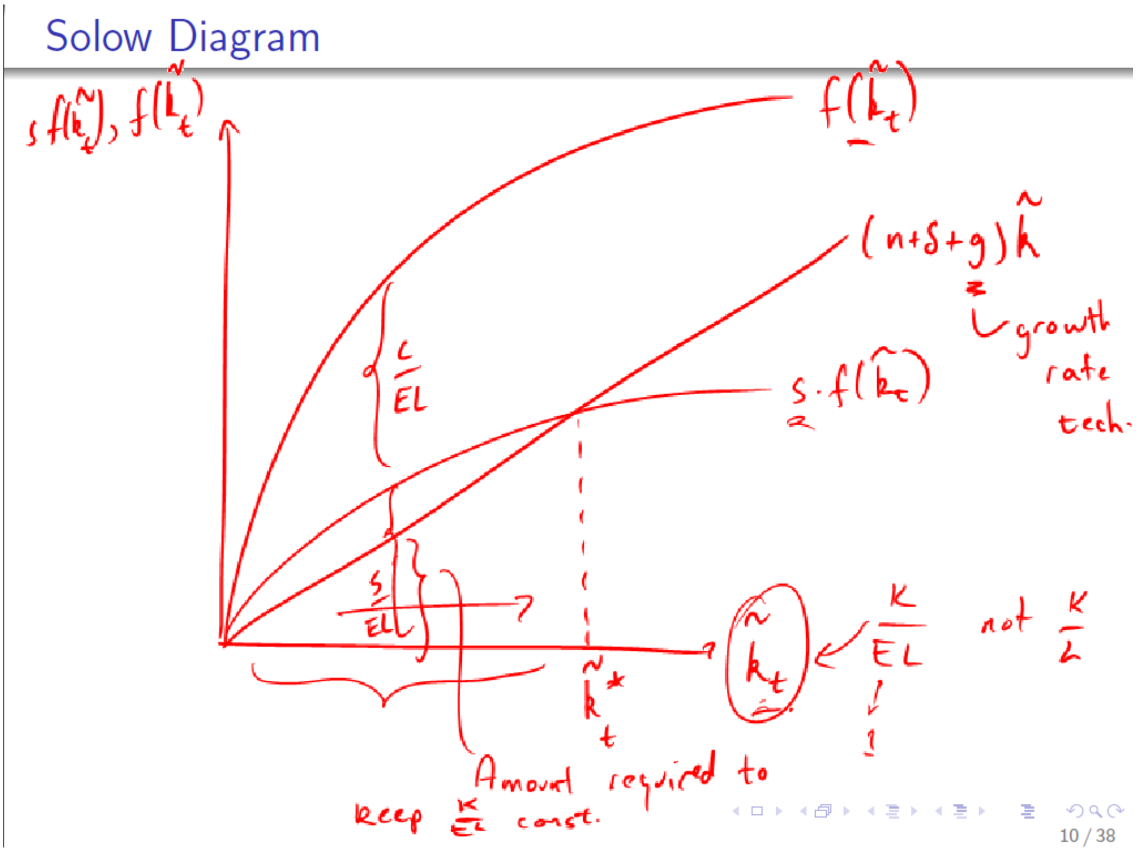
Exchange Rates

- Nominal exchange rate: determines cost of domestic currency in terms of foreign currency
- Real exchange rate: price of bundle of goods, purchased domestically divided by price of bundle of goods, purchased in a foreign country $\left(\frac{P_d}{P_f/e}\right)$
- Purchasing price parity: nominal exchange rates adjust to equate prices of goods across countries; fails because of transportation costs and many non-traded goods
- Interest rate parity: nominal exchange rates adjust to equate the returns on assets across countries interest rates and exchange rates

Exchange Rate Regimes

- Floating exchange rate: value of currency determined by supply and demand
- Fixed exchange rate: value of currency determined by government regulation either via limitations on trade or buying and selling of assets
- Currency unions: can think of as a fixed exchange rate among a set of countries and flexible with respect to other countries
- Benefits: Increased trade, reduced currency volatility and disciplined monetary policy
- Costs: Loss of independent monetary policy; these are smaller if regions are more similar and should share similar monetary policy
- Trilemma of international finance: it is impossible to have a simultaneously achieve a stable value of currency, monetary policy independence and free flows of capital
- Floating exchange rates give up a stable currency, while fixing an exchange rate requires either capital flow restrictions or giving up independent monetary policy

Graphs



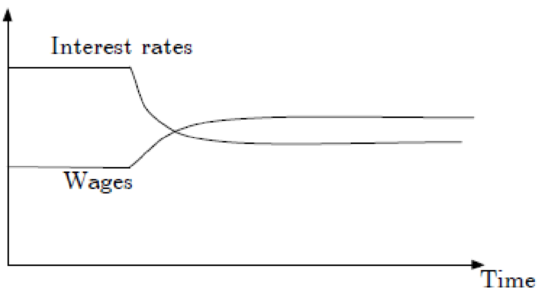


Figure 2: Transition paths, capital per worker, wages, interest ratest

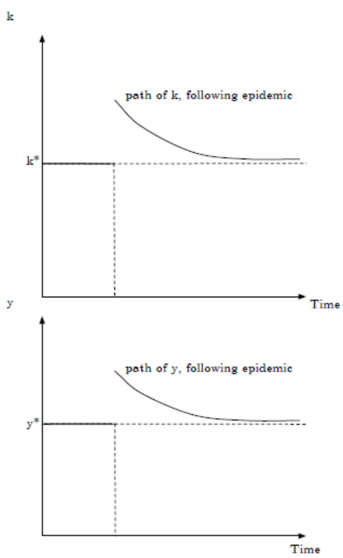
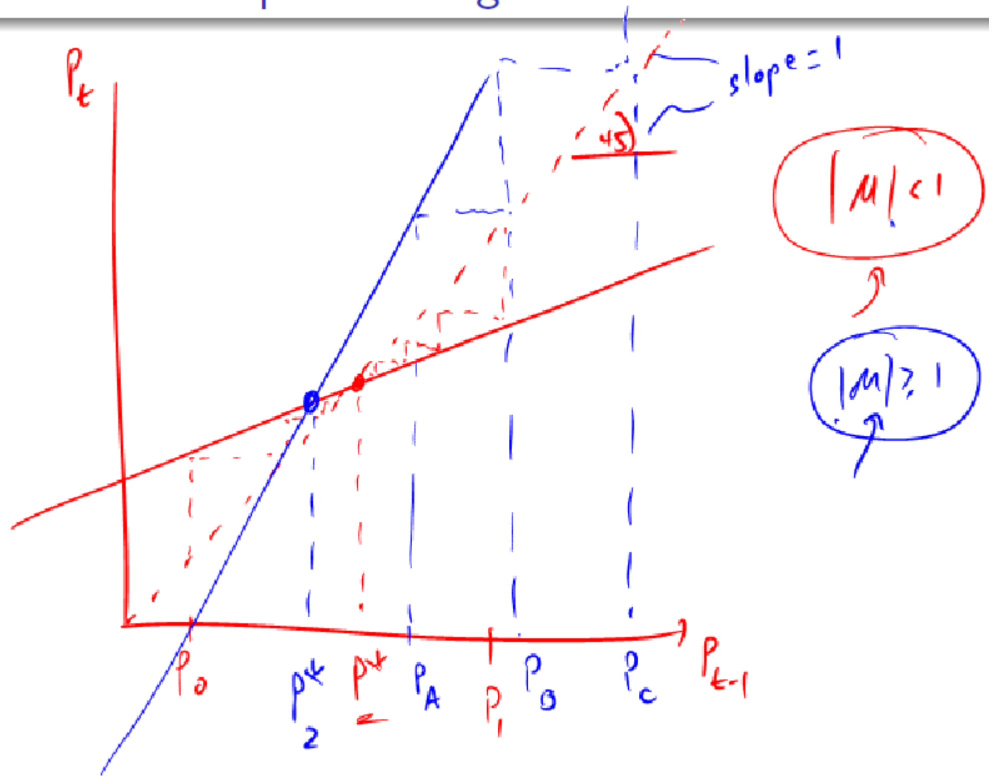


Figure 2: Long run effect of epidemic on capital and output per worker

Intuition: Difference equation diagram



Cobweb Model: Diagram